THE MATHEMATICS TEACHER

Volume XLI OCTOBER · 1948 Number 6

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OFFICIAL JOURNAL OF THE
NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS
525 WEST 120TH ST., NEW YORK 27, N.Y.

Printed at Menasha, Wisconsin

Entered as second-class matter at the post office at Menasha, Wisconsin. Acceptance for mailing at special rate of postage provided for in the Act of February 28, 1925, embodied in paragraph 4, section 412 P. L. & R., authorized March 1, 1930.

THE MATHEMATICS TEACHER

Official Journal of the National Council of Teachers of Mathematics

Devoted to the interests of mathematics in Elementary and Secondary Schools

The National Council has for its object the advancement of mathematics teaching in elementary and secondary schools. All persons interested in mathematics and mathematics teaching are eligible to membership. The membership fee is $3 per year and entitles each member to receive THE MATHEMATICS TEACHER which appears monthly except June, July, August and September.

Correspondence relating to editorial matters, subscriptions, advertisements, and other business matters should be addressed to the office of

THE MATHEMATICS TEACHER
525 West 120th St., New York 27, N.Y. (Editorial Office)

Subscription to THE MATHEMATICS TEACHER automatically makes a subscriber a member of the National Council.

SUBSCRIPTION PRICE $3.00 PER YEAR (*eight numbers*)

Foreign postage, 50 cents per year; Canadian postage, 25 cents per year. Single copies 40 cents. Remittance should be made by Post Office Money Order, Express Order, Bank Draft, or personal check and made payable to THE MATHEMATICS TEACHER.

PRICE LIST OF REPRINTS

	4pp. 1 to 4	8pp. 5 to 8	12pp. 9 to 12	16pp. 13 to 16	20pp. 17 to 20	24pp. 21 to 24	28pp. 25 to 28	32pp. 29 to 32
50 Copies	$2.88	$4.66	$7.30	$7.59	$9.85	$11.35	$13.11	$13.97
100 Copies	3.51	5.58	8.51	9.37	11.90	14.03	16.39	17.60
Additional copies per C.	1.21	1.84	3.05	3.62	4.14	5.41	6.61	7.25

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Plates printed on one side: For first 50 plates, $2.30; additional 2¢ each. Printed on both sides, for first 50 plates, $3.45; additional 3¢ each.

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The New President of the National Council of Teachers of Mathematics

THE MATHEMATICS TEACHER

Volume XLI Number 6

Edited by William David Reeve

Meaning in the Junior High School*

By Virgil S. Mallory

State Teachers College, Montclair, New Jersey

There is need for better teaching; need for teaching children so that the mathematics they study will be understood and will function in their lives. The teaching and the understanding must not pose hurdles in later years. Teaching and understanding must be so clear that they can be recalled to help the child when he forgets. We know that teaching is done by teachers. Today, because of the increased birth rate, of low salaries and high living costs, the shortage of teachers is appalling. A large per cent of teachers in our elementary and junior high schools hold substandard licenses. With a doubled birth rate, the outlook for the next eight or ten years is far from bright unless more young people are attracted to the teaching profession. Unless such steps are speedily taken, the young generation will fare much worse than any present survey might show.

Despite all these handicaps, with the usual optimism of teachers, we shall stress today the need for expert teachers, teachers who will not be satisfied with purely mechanical devices or mechanical teaching; teachers who will make little children *understand* arithmetic.

There is quite general agreement among educators with respect to most of the theses I shall present. Differences of opinion arise with respect to the method of attaining desired results.

1. *In order for mathematics to be useful to the child, it must have meaning.*

This meaning must have two aspects: mathematical meaning, so that the child can correctly perform the operations and processes; and social meaning, so that he can apply the mathematics to problems he needs to solve and thereby have functional competence in mathematics.

2. *Drill or practice in processes in mathematics is necessary in every grade from the first through college mathematics.*

Drill that is not accompanied by understanding, both mathematical and social, is of little use. Heterogeneous drill in arithmetic, given without diagnosis of a pupil's difficulties, without any attempt to locate the points where explanation is needed, may be worse than useless. It may do harm. Such drill gives the pupil practice in repeating the errors he has previously made. In no way does it effect a cure for his deficiencies. In a certain school test results had shown that a large number of high school pupils were below standard in arithmetic computation. No diagnostic tests were given to locate specific indi-

* Paper read at the Annual Meeting of the National Council of Teachers of Mathematics at Indianapolis in April 1948.

vidual difficulties. The pupils were placed in classes and given mimeographed drill work on the fundamental operations long difficult addition, subtraction, multiplication, and division problems with whole numbers, common and decimal fractions. The drill work, made by the mathematics teacher, showed no understanding of how arithmetic is taught in the primary and intermediate grades. There was no attempt to begin with basic understanding. As a method of improving a pupil's skill in arithmetic, it was worse than useless. The child could learn more mathematics listening to Tom Mix on the radio than he could in this mathematics class, for no attempt was made in the course to impart understanding or to correct deficiencies in mastery of skills or in meaning. The drill work was simply continued practice in making errors.

The correct procedure should have been: determine the specific difficulties each pupil has, whether in primary facts, the fundamental operations, operations with common or decimal fractions; explain each by going back to concrete illustrations as though the topic had not been explained before; give each pupil practice to fit his needs, motivated because the pupil knows that he is correcting previous faults.

3. *Meaning and understanding must be defined in terms of the child's actual reaction, not in terms of what his reaction should be.*

No matter how fine the teaching method or how elaborate the teaching aid, unless they give the child a true concept of the arithmetic he is studying, the device or teaching aid is not of great value.

At a syllabus committee meeting some years ago one of the members suggested a device for making signed numbers meaningful. The device was not familiar to some members of the committee and was explained to them. When they finally understood, all agreed that the device was more complicated and difficult to understand than the concept it attempted to make clear. All of us agree that the child must understand place value in our notation for writing numbers. We must be certain that the methods we use to get understanding do not result in confusion.

The abacus is a useful teaching aid. We are not concerned here with the historical developments of the abacus: the Chinese S'uan p'an, the Japanese Soroban, or the Russian abacus. We are concerned with beads on a wire to help children understand counting and place value. Beads on horizontal wires can be used for counting. Beads on vertical wires can be used to show place value, borrowing in subtraction, and carrying in addition. But despite the usefulness of the abacus, the abacus will not automatically make the abstractions that are necessary for the child to make when he adds and carries or when he borrows and subtracts. We do not want him to acquire the notion that yellow beads are ones, blue beads are tens, red beads are hundreds. He must know more than that the second vertical string represents tens. Writing a symbol 5 in the second column to represent 5 tens is more abstract than counting 5 beads on the second wire.

There is no sure road to understanding and meaning merely by the use of teaching aids. The teacher must be certain that those concepts which are developed by the teaching aid are those which lead to a better understanding of mathematics. It is true that a motion picture *may* show the uses of geometric form better than the teacher could. The teacher must not forget, however, that the best teaching aid is the simple device readily at hand in the classroom. To show elementary principles and to give mathematics that reality necessary for competence in the use of mathematics: use scissors; use rulers and tapes for measuring; fold paper; show that a cylindrical and a conical cup filled with sand give the relation between the volumes of these solids, and then, through induction, reach conclusions.

For *reality* is necessary to reach the final

stage of concreteness in teaching mathematics. And no matter how interesting the story in the textbook, its interest is no guarantee of reality to the child.

Some years ago I was doing my homework in 7th grade arithmetic. My 11-year-old daughter was supposed to assist. She was bright and active and interested in *so* many things. It was quite a feat to have hooked Daddy for the homework. More time would be left for more interesting things. The problems were rather an interesting configuration of problems about the expenses of a family taking an automobile trip. Ruth, at my request, read the first problem, and then waited for my suggestions, for she had none. I had her read the story again and then, tell me the story in her own words. It was evident that items in a trip important to an adult were not important to her. For her there was no reality in the story. Finally, exasperated, she said: "We're just wasting time. What shall I do: add, subtract, multiply, or divide?"

Jack was a bright 8th grade boy. He was skillful in computation and was always one of the first to have the correct answer. The class was solving, from the text, the problem: "A cylindrical tank is 6 feet in diameter and 4 feet high. How many gallons of water will it hold?" Jack soon had the answer: 0.848 gal. "Approximately how much?", I asked. "About a gallon." "How many quarts?" "A little less than 4." "How large is the tank?" Jack read his problem again, with no appreciation of the incorrectness of his answer until I asked him to show me, with his hands, the size of the tank. Then he properly placed the decimal point. Until he began to measure out the size of the tank, the book problem had no reality for him

The teacher asked Harry: "If bananas cost 5 cents each, how many bananas can you buy for $1.25?" "250," said Harry. "All right," said teacher, "Take this $1.25 and go to Tony's fruit stand and get me 250 bananas." "I can't get that many," said Harry. "I thought it was just a problem. I didn't know you meant business." In none of these examples did the problem proposed have reality.

Children must understand enough about place value to understand our decimal notation and to be able to intelligently place figures in computation. We must be careful that we do not hopelessly confuse them by seeking an understanding beyond that possessed by adults or needed by adults. Children need to understand that our place value notation has units, tens, hundreds; that zero is not only a place holder, but also a number that shows how many there are in that place; that in adding a column of figures, if the sum of the units is 23, we have 2 tens and 3 units and we carry the 2 tens to the tens column; that in subtracting 13 from 32, we cannot take 3 units from 2 units, so we borrow 1 from the tens column and thus have 10 units to add to the 2 units we had before. If the child will be helped by writing the figure to be carried there is no harm done. Most accountants do so for aid in checking. If the child crosses out and corrects the number from which he borrows, let him do so. We are not so nearly concerned with speed as we are with understanding.

In multiplying the child can grasp the fact that 3 units times 5 units is 15 units, or 1 ten and 5 units; that the 1 ten is carried to the ten's column. To carry this place value analysis much further is to seek an understanding beyond not only the pupil's powers but beyond those of most adults. What is 3 hundreds times 5 hundreds? Four ten thousands divided by 2 hundreds?

In division the situation is particularly difficult. The algorism for division is difficult in its own right. There is no excuse for adding difficulties. Suppose we find how many times 3 is contained in 132. Certainly the child must know that 132 is 1 hundred, 3 tens, and 2 units. But these facts, this knowledge, can well be a hindrance to learning if applied when it is

not needed. One method of showing the division of 132 by 3 might be this:

$$3\overline{)132}$$

"I cannot divide 1 by 3. So I examine 13. Since $3\times 4=12$, $13\div 3$ is 4."

"Since I am dividing 13 by 3, the quotient 4 is written above the 3 of 13." The child then multiplies, subtracts, compares, brings down, and completes his division, placing the last figure of the quotient above the last figure of his divisor. The answer is 44. The child checks by multiplying 44 by 3. Knowledge of place value has not been used to confuse him. The method is straight forward and simple.

Let us consider another method of presenting this problem, a method proposed by some educators. "We are to divide 1 hundred, 3 tens, and 2 units by 3 units. But 1 hundred cannot be divided by 3 units. (Of course we know that 100 can be divided by 3. The statement of truth is more subtle than that. It is: 'The *number* of hundreds cannot be divided by the *number* of units.' I doubt that many children will see the distinction.) "So we examine 1 hundred and 3 tens, and call this number 13 tens." (Another skill is called for here which in no way helps in the development of the algorism nor in giving the child understanding of division.) "Then 13 tens divided by 3 units is 4 tens." (Will the child know that the result is tens, and not units?)

"Multiply 4 tens by 3 tens and get 12 tens." (Will he know it is tens and not hundreds?) "Write 12 tens as 1 hundred and 2 tens. Subtract and get 1 ten. Bring down 2 units. Then 1 ten and 2 units is 12 units. 12 units divided by 3 units is 4 units." (Even though the child knows the multiplication fact $4\times 3=12$.)

It is possible that the child will become confused by the language, hopelessly confused by all of these units, tens, and hundreds (who wouldn't be confused) as was the child who divided 28 by 7 as follows:

"28 is 2 tens and 8 units. I divide the units first. 8 units divided by 7 units is 1 unit which I place in the units' column above 8. 1 unit times 7 units is 7 units which I subtract from 8 units and get 1 unit. Bring down the 2. Then $21\div 7$ is 3, which I place in the ten's column since 21 is 2 tens and 1 unit."

$$\begin{array}{r} 31 \\ 7\overline{)28} \\ 7 \\ \hline 21 \\ 21 \\ \hline \end{array}$$

The confusion in this development is even more evident when there is a two figure divisor as $23\overline{)115}$. Again we have the subtle statement: "1 hundred cannot be divided by 2 tens. Then call 1 hundred and 1 ten, 11 tens. 11 tens$\div$2 tens is 5 units." The use of "tens, hundreds, etc., as concrete numbers instead of as abstract numbers, necessary by this method as in dividing $200\overline{)6800}$ where the child must answer the question: 6 ten thousands divided by 2 hundreds is what? (Can you answer this, off the bat?) shows that there is little of real meaning and understanding to be transferred for later use by the child. And division by 3 figure numbers is even worse. But it is required in the junior high school in problems in mensuration, making graphs, and in the use of money.

Certainly place value must be taught, but always within the child's ability to understand. He must not be asked to make generalizations and abstractions that would puzzle many adults. And the learning, the meaning, must be easily recalled in later years.

4. *Every teacher of arithmetic should know well how arithmetic is taught in the lower grades.*

This statement applies not only to junior high school teachers but also to teachers in the senior high school. It is a well known fact that skills in any field deteriorate through lack of use. An adult would not want to take an examination now in some of the fields he mastered years ago in his undergraduate days at college. Dr. Thiele, in a study in Detroit, showed that in high school a pupil deteriorates in arithmetic skills, unless he has

current use of arithmetic, as he gets farther away from the eighth grade. In a similar study at Montclair, the same results were obtained with pupils in the college preparatory course, showing that college preparatory mathematics does not provide the practice in number needed to maintain arithmetic skills.

If a pupil in any grade has forgotten facts or processes in mathematics, the only recourse is for the teacher to explain them, giving them meaning, and reteach them at that time. No valuable end is served if the teacher wrings his hands and tries to place the blame on earlier teachers, on the parents, or on Providence. This reteaching, this putting meaning into forgotten processes, can only be done by a teacher who has studied how arithmetic can be taught with understanding.

5. *At all stages in the teaching of arithmetic the development must begin with the concrete then go to the abstract and finally be applied in the concrete situations of real life.*

Plato gave good advice when he said that to teach children how to count, begin with objects familiar to them: apples, their toys, balls. So we begin with concrete objects. The child counts pencils, chairs, children, and beads on the abacus. When he learns to say the numbers "one, two, three, four, five" by rote, he does not necessarily know how to count. He must be able to place these number names in one-to-one correspondence with the five apples he is counting. Until he has learned to abstract so far as to place his finger on one apple and say "One," on a different apple and say "Two," and so on until each apple has been associated with a different number name, he does not know how to count.

There is still a further abstraction that the child must be able to make in counting. He must learn that the cardinal number *five* can be used to determine the count of any kind of object: candies, chairs, children, money, or just things. Similarly with the ordinal number "fifth." With this high degree of abstraction in so simple a process as counting, it becomes plain why teachers must understand the difficulties involved and must realize that there can be no mastery of arithmetic unless the child understands.

We begin with concrete objects and situations and continue with them until the child can make the abstractions required. The task is not complete until the child is able to apply what he has learned in concrete every day life situations. He must begin with the concrete and end with it. In between he must abstract.

6. *Understanding is hindered by the use of words or devices that conceal mathematical meaning.*

Let us take the word "cancel" as an example. There is no excuse for introducing the word "cancel," a word which may mean either subtraction as in 6–6 where the 6's cancel each other and give zero or may mean division as in 6/6 where the 6's cancel and give 1. There are enough abstractions in arithmetic without the use of confusing terms like cancellation. $6-6$ is equal to zero. There is no cancelling. There *is subtraction.* 6/6 is equal to one. The numerator and denominator are each divided by 6.

Bright Barbara brought her homework for dad to examine. She had changed 250/450 to per cent and had 50% for an answer. She explained how she did it. "First I cancel the zeros and get 25/45. Then I cancel the 5's and get 2/4 or 1/2. That is 50%."

If one is going to cancel, that is bright work, showing that the child could abstract, and could even generalize in an unusual situation.

Tom in algebra reduced x^3+y^3/x^3-y^3 and got 1 for an answer. He explained, "You cancel the x^3's, and the y^3's." "Yes" I said but "how do you get 1? There is left $+/-$." "Well, the horizontal lines cancel and 1 is left."

Again, teachers sometimes use a gallows-like device for adding and subtracting fractions. The common denominator of

the fraction is placed on the roof, and the numerators are segregated inside. The only real claim that can be made for the device is that it effectively conceals meaning and that any connection with understanding is only coincidental. *Keep the child close to basic meanings.* No time is saved if a process or device must be remembered by main force because there is no meaning in it.

7. *A critical survey of materials of instruction in grades 7 and 8 should be made to be sure that all have social meaning for the child.*

A brief list of the items that should be considered, in the reverse order of their value, are: stocks and bonds, insurance, extensive treatment of mortgages, taxation, and perhaps to a lesser degree, buying a car or a home, installment purchases, and interest; on the mathematical side there might also be considered the value of teaching volumes and surfaces of the sphere and cone to children in the 7th and 8th grades.

I have suggested that we re-examine these topics to determine whether or not there is anything in them that has social significance or useful mathematical value for junior high school pupils. I have taught these topics in every grade from grade 7 to college graduates.

We teach such a general course to juniors (not mathematics majors) in the College at Montclair. I have sought consistently for that degree of motivation which would make some of these topics have reality for the students. I never found that desirable degree of maturation in the 8th grade, the 9th grade, or the 12th grade. I never found it for college students until I slipped into the notice of an extension course on Social Uses of Mathematics, mainly for junior high school teachers, the statement "Newly-weds will be interested." And they were there in numbers: girls married a few months, girls married a few years. There were vital questions: "What kind of insurance? Why? What will it cost? Why is it sometimes better to borrow at a bank than to pay installment charges? We have no securities; how can we borrow money to the best advantage? What are credit unions? Where can they be found in our locality? What are co-ops? Is this a good plan for buying a home, a car?" They had no interest in stocks and bonds. Their interest in taxation was entirely on how they could save money in their tax payments. There was motivation and interest that could not have been obtained in any 7th, 8th, 9th, or 12th grade class. Still, American children should not leave school without some information about some of these topics. *Information* not necessarily *computation.* Certainly if in every 12th grade class in every high school there was offered for all students, college preparatory as well as general, a course in Consumer Mathematics, the problem of eliminating some of this subject matter in grades 7 and 8 would be more easily solved.

Certainly no junior high school student can be interested in finding the broker's bill for buying or selling 68 shares of A.B.C. stock at 39 7/8 with a brokerage fee of 1/8%, and a tax of so much per share. But he may be interested in the story of the boys who joined in renting boats to a summer colony, and who issued shares to themselves in ratio to the money they advanced. He *may* be interested if the teacher is a convincing story teller. I always tie such stories to my personal experiences. I start out: "At Cobleskill, where I have my farm there is a wonderful lake up in the hills. It is called Boucks' Bowery Lake. Now Boucks is a man's name, one of the early settlers. Bowery means farm." And so I spin the tale, with with poet's license, spending too much time perhaps in describing the terrain, the fishing, until the children live with me the wonderful delights of the lake at Boucks' Bowery. And then I tell about the neighborhood boys, a personal note about each to make the tale real, and their scheme to buy boats, rent them to sportsmen, divide the profits. Before we are

through the eighth graders know all they need to know about a company, shares of stock, and bonds; about shares that sell below their value and those that sell above it. For these basic concepts have all entered into the business the boys ran. The teacher who presents this material needs to be more than a teacher of mathematics. He needs to be an artist and perhaps, a consumnate liar. It is, however, all in a good cause.

8. *The mathematical concepts developed must correspond to the pupil's maturation.*

We have talked a great deal about grade placement, many times with our attention focused on the logical development of mathematics and not on the child's development. As has been often said, we should arrange mathematics to the child's needs, not break down the child so that mathematics may be more elegant.

We must appreciate the fact that through the ten or twelve years the child is in school there are certain concepts that should be so integrated in the child that he may live a rich, full life.

Rather than set grade placement as an arbitrary limit on facts and processes, we must consult the child's maturation with respect to concepts and ideals. The emphasis on the child, and on his ability to develop rich concepts, turns our attention from *mathematics* to the *child;* from too great emphasis on skills and processes, to the development of appreciations and concepts. One topic not concerned with the development of arithmetic skills, but with appreciation of life about the child, is geometric form. Because of the present emphasis on facts, skills, and processes in arithmetic, geometric form is neglected in our present course of study.

In any philosophy that considers education to ages 16 or even 18 in some states, beyond the bread-and-butter line for any subject, geometric form must be considered. Certainly we do not expect the 8-year old child to have as rich concepts of these forms as we expect the 14-year old child to have. If we build appreciations, integrate these with drawing and art, and emphasize geometry in form, and later in size, by the time the child is in the junior high school, through drawing, constructing, cutting, and folding he will have far richer concepts and will develop habits and skills in seeing relations, in drawing conclusions from evidence, in appreciating beauty, and in creating it through the use of such forms.

The ability to make estimates is important. Not alone the ability to estimate size, but the ability to estimate the correctness of an answer, to determine the position of the decimal point without counting the number of places in multiplication and division. To develop the habit of making estimates requires extensiveness in time for its development. A teacher in the junior high school can go further and faster in developing this skill if the child has been accustomed to doing so in earlier grades. But the maturity of the child, the backlog of mathematical skills previously developed, the ability to multiply and divide by powers of 10 or multiples of such powers, will determine success in any such undertaking. Hence, there are few estimates that the third grade child can make. In succeeding years others will be added but they will be circumscribed by the understanding the child has of mathematical relationships. They should never be pushed to the point where the child makes unmeaning *guesses* instead of thoughtful *estimates.*

9. *Since the child has completed practically all of the facts, skills, and processes by the time he enters the 7th grade, the junior high school teacher has the task of making him sure in his ability, and certain in his understanding.*

As pointed out previously, if the child fails to understand any detail in the work, the only cure is to teach him then and there, going back to concrete illustrations, and emphasizing meaning and understanding. Those topics he has learned most recently will be the ones requiring most teaching: operations in common fractions,

placing the decimal point in multiplication and division of decimals, and the uses of per cent are the old topics in which he will need review. Extensive work in direct and indirect measurement should be given, using simple instruments made in the school wood shop by the pupil. If he makes his transit, he will understand what he is doing and the teacher can reach objectives of careful workmanship, respect for accuracy, appreciation of functional relationships, a recognition of the fact that a small error in construction may make an unbelievably large error in computation.

An eighth grade class was measuring the height of the flagpole. They had made enough school transits in the wood shop to supply one for every four pupils.* The janitor had been warned not to tell the pupils how high the flag-pole was. All out door measurements had been made and the pupils were making scale drawings in a supervised study class. There was much interest and excitement. From the far corner of the room Jeff shouted (we have no discipline in my classes as long as we are learning mathematics) "Mr. Mallory, look at this," and so I hurried over to have Jeff tell me of the wonderful discovery he had made. "Look! The angle is 38°. I copied it 39°. And there is a difference of 10 ft. in the height of the flagpole. Can you imagine it. And it is all because the protractor is not accurate at both ends."

What a discovery! What appreciation of accuracy in measurement, of functional dependence. Enough to gladden the heart of a teacher for many a long day.

10. *We cannot allow our program to entirely ignore the brighter student.*

In examining the stepped-up course of study in arithmetic and the decisions made about what a kindergarten child does and does not know, it is evident to any observer who knows the ability of the upper 20% of these children, that we are placing our course of study at the level of the moron. Thus we neglect that group in the upper intelligence brackets to which we have been inclined to confer the title *future leaders*. It is true that they may, by and large, lead in science and literature. It is not so certain that they will be the leaders in commerce, wealth, or in politics. However, we do owe these better students a duty in the teaching of mathematics, a duty that we can fulfill in a number of ways in the junior high school by teaching (1) Short cuts in computation; short ways of multiplying or dividing by powers of ten or multiples of powers of ten. Ease in estimating results is a valuable end product. (2) The method of short division by a one-figure divisor. It is absurd to expect a bright child to do otherwise. (3) Multiplication and division by the use of aliquot parts, one of the finest ways to teach mathematical meaning. (4) The expression of geometric conclusions from a concrete examination of geometric figures followed by induction with a full appreciation of the fact that such conclusions need proof, and that induction or visual examination does not constitute proof. (5) A short journey to deductive proof, so wisely conducted by the teacher that the bright pupil will be thrilled by the prospect opened to him- This honor work must emphasize reason. ing and may even include a short unit in deductive reasoning. For example:

First, have the class draw triangles and, by measuring, find the sum of the angles. While answers may vary, a reasonable conclusion is: "In the different kinds of triangles we have measured today, the sum of the angles *seems* to be 180°."

Second, have the class draw triangles, cut them out, tear off the angles, and paste them on a sheet of paper adjacent to each other, A reasonable conclusion is: "As far as we can judge, the sum of the angles in all of these triangles *seems* to be a straight angle or 180°."

So all of the class has reached a tentative conclusion by induction. For the better pupils we can reach a real *proof* by

* See directions for making school transits in Mallory, "*Mathematics for Everyday Affairs*," Benj. H. Sanborn and Co., Chicago, Ill. Pages 453–56.

deduction. We shall not necessarily require them to reproduce it but they can be made to appreciate it.

1. Define a straight angle as an angle of 180°.
2. Define alternate interior angles.
3. Define parallel lines as lines which have alternate interior angles equal.
4. Through the vertex C of triangle ABC, draw l parallel to AB. Then *prove* that $A+B+C=180°$.

In all of this development there must be a real understanding of the difference between *assumptions* and *proof*. We are not at this time concerned with child's ability to reproduce a proof. We *are* concerned with his *appreciation of* proof.

To summarize: (1) We must have *meaning* and *understanding* in the teaching of msthematics and *reality* in social problems. *Meaning, understanding,* and *reality* must all be in terms of the pupil and his appreciations. The teacher's definition of *understanding* is futile if it results in confusion for the child. The book problems may seem ever so real to the author of a text but are useless unless the teacher breathes the breath of life into them. The teacher of mathematics in the junior high school has a challenging task in three respects: (1) He must correct previous errors in understanding by going back in every case to concrete illustrations which lead to abstractions and which finally are interpreted into everyday situations. Thus he secures functional competence for his pupils. (2) He must introduce social situations in such a way that his pupils will live the situations. Hence he must be a scholar, a practical man, an artist, of no mean proportions. (3) He must breathe the breath of reality into book problems. Only in this way will the children in his care develop that social and mathematical meaning which means functional competence.

A Summary of Research and Investigations and Their Implications for the Organization and Learning of Arithmetic*

By H. Van Engen
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It is obviously impossible to give a complete summary of arithmetic research in the brief time allocated for this paper. Periodic summaries of the research in arithmetic are available in such magazines as the *Elementary School Journal* and in the *Review of Educational Research.* In many cases these reviews are accompanied by comments which point out the lacunae left by the uncoordinated attack on research problems in arithmetic. In many instances, these comments are particularly pertinent to the topic of this paper. It will be my purpose to bring in many of the points of view expressed in these well-known summaries and bring them to bear upon the organization and learning problems in the field of arithmetic.

At least two contributions, in this general area cannot be overlooked. Buswell gave a paper at the Conference on Arithmetic at the University of Chicago in the summer of 1946 on "The Outlook for Research in Arithmetic."[1] Brownell wrote on "The Frontiers in Educational Research in Arithmetic"[2] in the *Journal of Educational Research.* These papers touch on today's topic but, from a different point of view. Buswell and Brownell were mainly interested in stimulating more research on the problems of arithmetic by highlighting the outstanding problems in the field.

With the afore mentioned summaries of research, it would be rather pointless for me to spend my time giving another summary. Instead, I am going to give you one man's opinion on the implications of research for the organization and learning of arithmetic as it affects the teachers of arithmetic, and, in passing, I will indicate where in I think the research studies, as a whole, have been lacking. To bring the paper down to assigned time limits, I will use only the studies that have been published in comparatively recent years, and pointed specificially at arithmetic learnings.

To facilitate organization, it has been found necessary to classify recent research studies in six categories. Some of these studies are not easily categorized because they deal with two, or more, of the main headings I have set up. Because of this difficulty, I hope you will give me the privilege of classifying a study under, say, Readiness when it could also be classified under The Learning Process.

I. Meaning. The word "meaning" has found its way into the language of the arithmetic teacher, in the past decade, with ever increasing frequency. Like many words, which are used frequently, there seems to be evidence that all teachers really do not interpret the "meaning" of meaning in the same way when applied to arithmetic. As evidence, read the protests found in the literature, against those teachers who say they are teaching arithmetic meaningfully, but according to numerous authors, these same teachers are only good "drill sergeants." Briefly described it is often heard said that "teachers give lip service to meaning."

Now part of this difficulty, I am quite

* Paper read at the annual meeting of the National Council of Teachers of Mathematics at Indianapolis in April 1948.

[1] Buswell, G. T. "The Outlook for Research in Arithmetic." Presented at the Conference on Arithmetic held at the U. of Chicago, 1946. U. of Chicago Press, p. 35.

[2] Brownell, William. "Frontiers of Educational Research in Arithmetic." *Journal of Educational Research,* 40: 373, January 1947.

sure, is due to the fact that the "meaning" of meaning has never received proper attention in the literature. It is true that there are many excellent treatments of the meaningful teaching of, say, the additions facts. But why is one method more or less meaningful than another method? What is this so-called "meaning"?

There have been a few attempts to define the phrase "meaningful arithmetic" so that the elementary school teacher can understand it. One of the best attempts that I know of was written by Anita Riess in the *Elementary School Journal*.[3] In this article Miss Riess gives a behavioristic definition of meaning which is applicable to the elementary processes of arithmetic. However, more attention must be given to the "meaning" of meaning. It should receive more attention in books on methods in elementary arithmetic just as the learning process now is an essential part of every complete treatment of methods in arithmetic.

And what about test for meaning? This is an exceedingly fruitful field for further study. There are a number of research studies "in the air," so to speak, but not nearly enough has been done in this area. Sueltz has an article in the *Elementary School Journal* on "Measuring the Newer Aspects of Functional Arithmetic."[4] Sueltz has attempted to develop a test to measure understanding in arithmetic and to measure judgments as applied in quantitative situations. The administration of the test has shown that pupil responses are not at all satisfactory. Assuming the validity of the tests, Sueltz results would seem to indicate that as classroom teacher we fall far short of attaining meaningful results.

Here, then, is an area in which there is much need for an adequate definition if the teacher of arithmetic is to get a generalized idea of what it means to teach arithmetic meaningfully. Furthermore, much work must be done in learning how to measure some of these newer and more intangible outcomes of arithmetic instruction. In the meantime, the teacher of arithmetic can best use individualized pupil activities to measure the pupil understanding of arithmetical processes.

II. The Learning Process and Methods. As with meaningful arithmetic, the nature of the learning process in arithmetic has been commanding an ever increasing amount of attention. The reason, of course, is rather obvious if one is moderately acquainted with the work that has been done in this area. The two go hand in hand in a modern instructional program.

The changing emphasis in research in arithmetic can be roughly gaged by a few statistics. The *Review of Educational Research* in 1942 does not mention the learning process as a major subdivision in its survey of research in arithmetic. In 1945, the *Review of Educational Research* used "The Nature of the Learning Process" as a major subdivision in its classification of research studies. *The Elementary School Journal* report at least three studies that could be classified as learning studies for each year from 1942 to 1945 inclusive. In the review for 1945–46 there are eight studies that could be classified as research on the learning process in arithmetic.

The most outstanding study in this area covers a rather well fought-over battle field. The question as to whether the equal additions method or the decomposition method should be used has been discussed, seemingly, for some centuries. However, it remained for Brownell[5] to apply the modern concept of learning arithmetic to this problem to produce something which has definite implications for the organization of learning activities in arithmetic. Very briefly stated Brownell found that

[3] Riess, Anita. "The Meaning of the 'Meaningful Teaching of Arithmetic.'" *Elementary School Journal*, pp. 23–32, September 1944.

[4] Sueltz, Ben Albert. "Measuring the Newer Aspects of Functional Arithmetic." *Elementary School Journal*, Vol. 47, pp. 323–30, Feb. 1947.

[5] Brownell, William A. "An Experiment On Burrowing in Third-Grade Arithmetic." *Journal of Educational Research*, 41: 161–71, Nov. 1947.

the decomposition method of teaching subtraction was better than the equal additions method provided that rationalization was one of the ends of instruction.

This result definitely indicates that it is imperative for teachers of arithmetic to consider the end product of learning. If the answer to a number fact is all that is necessary and desirable then one course of instruction seems feasible. If the teacher is interested in broad generalizations, understanding and fundamental knowledges then another course of instruction needs to be followed. In other words, it is not possible to "just teach arithmetic." The teacher must decide what ends she wishes to achieve and choose instructional procedures which best achieve these ends.

D. Banks Wilburn reported a study in the *Elementary School Journal* on a "Method of Self-Instruction for Learning the Easier Addition and Subtraction Combinations."[6] This study is significant because Wilburn found that children could be taught a method of studying groups which relieved the busy teacher of some of the routine of presenting all the combinations either individually or by the well-known groups based on the inverse operations. Wilburn found that studying groups of, say 5, first, and mastering all combinations of five, then giving the child another group of objects, to use for mastering the combinations for that group of objects by means of a definite learning sequence, produced very good results. The implications as to method are clear. Children can be taught methods of self-instruction. Its implications as to the organization of learning activities in this area are also easily seen. It raised the question as to whether teachers should not consider the study of groups as a more powerful method of studying combinations than the "unit of four" method.

Another study reported by Brownell in the *Journal of Educational Psychology* entitled "Accuracy and Process in Learning"[7] calls attention to the fact that the evaluation of learning should include some evidence of the degrees to which children actually grow in power of quantitative thinking. Here, again, is a call to reevaluate the outcomes of instructions. But even more than that. Note that Brownell placed emphasis on the rate of growth. The usual testing program does not take into account the rate of growth in arithmetic ideas, abilities, and generalizations. If the advocates of teaching arithmetic to children at a rate commensurate with his power to learn arithmetic, convince the educational world that their program should be put into wider practice then some index of rate of growth in arithmetic abilities would be an important factor for developing the program of studies.

III. Problem Solving. There is evidence that the teaching world is gradually changing its conception of what constitutes problem solving in arithmetic. The so-called problems found in the text book are now, at times, being called examples. There is a growing realization that at best the book problems are exercises in using the language of arithmetic and that very little problem solving activity may accompany the procurement of answers to a page of textbook problems. In trying to summarize the research in arithmetic on problem solving, one is reminded of Buswell's remark in his paper given at the University of Chicago Conference on Arithmetic in 1946 "Outlook for Research." "Research in arithmetic has more frequently been characterized by industry and perservance than fertility of ideas. . . . "[8] Of course, there are research studies showing the correlation of various abilities with that of getting the correct answer to a group of so-called problems. The results are interesting and valu-

[6] D. Banks Wilburn. "The Method of Self-Instruction for Learning the Easier Addition and Subtraction Combinations." *Elementary School Journal*, 42: 371–80, Jan. 1942.

[7] Brownell, William A. "Rate, Accuracy and Process in Learning." *Journal of Educational Psychology*, 35: 321, Sept. 1944.

[8] *Op. cit.*, p. 36.

able. There are vocabulary-problem solving studies leading to interesting results. However, the situation is rather confused. The confusion is due, in part, to the difficulty of getting at the heart of a problem situation and determining what children do in actual problem situations. Further progress in this area will probably be contingent upon progress in such areas of research as readiness in arithmetic, the learning process, meaning and general studies on the thinking process. More will be said of this in later sections of the paper.

IV. Readiness. For a complete review of research in this area one must turn to Anita Riess' "Number Readiness in Research."[9] This pamphlet contains an excellent summary of all that has been written on this problem, up to a very recent date. With such summary available, it would not be wise to do other than call your attention to just a few of the highlights.

The fact that readiness has received sufficient attention to encourage the publication of tests in arithmetic for measuring readiness is in itself noteworthy. This implies that teachers are beginning to recognize that one cannot begin teaching the addition combinations on a Monday without having given considerable forethought and some effort to collect data as to whether it is feasible to begin teaching the combinations on that particular Monday. A program of readiness for formal instruction in any process is essential in a modern school.

The literature has reports of readiness tests that have been published by such men as Brueckner and Souder.[10] Examination of these tests are well worth the time of any teacher interested in improving the instructional program in arithmetic.

On the more recent research studies in this area, one cannot overlook Doris Carper's study on "Seeing Numbers as Groups in Primary Arithmetic."[11] reported in *School Science and Mathematics.* Miss Carper made the observation that research has been largely limited to counting. She found that grouping was well within the powers of her subjects. She further calls attention to the fact that there is a lack of materials that the teacher can use to develop responses to groups as a preliminary stage to studying number combinations. Even many methods books overlook a "readiness to count program." Number ideas are complex. The first stage of a well rounded arithmetic program does not consist of having Johnny count objects. The recognition of group relations as a precounting stage should receive more emphasis.

The subject of readiness has received considerable attention in the lower elementary grades. Few studies have been centered on a readiness program for such ideas as comparison—subtraction and ration, fractions, decimal fractions, yes, and even negative numbers. In fact, textbooks, in the above named areas have almost neglected to consider readiness as an essential part of their program. A readiness program in common fractions has been developed in the texts in use today. Yet, its development has not been based on a program research such as the program of readiness for number combinations. A readiness program for such ideas as percentage is particularly noticeable by its absence. More must be done by the classroom teacher to develop a readiness for the ideas encountered on all levels of the elementary school. Until this is done each teacher can do a great deal of exploratory work, to the advantage of the children now in her classes, as well as to the advantage of the whole instructional program in arithmetic.

[9] Riess, Anita. "Number Readiness in Research." Scott Foresman and Co., Chicago, 1947.

[10] Brueckner, Leo J. "The Development and Valuation of an Arithmetic Readiness Test." *Journal of Education Research,* 40: 496, March 1947.

[11] Carper, Doris J. "Seeing Numbers as Groups in Primary-Grades Arithmetic." *Elementary School Journal,* 43: 166, Nov. 1942.

V. Social Uses of Arithmetic. Almost yearly a number of studies of the uses of arithmetic in various industries, or communities have been reported. These studies serve to keep "one's feet on the ground" and many are valuable contributions to the literature of research in arithmetic. Yet, they leave one with a feeling of indecision. If we as teachers of arithmetic would teach a meaningful arithmetic and attain such objectives as developing significant generalizations, how much of the social studies arithmetic would it be necessary to include in our course of study? Of course, some must always remain for the purpose of teaching the pupil that the processes of arithmetic are significant. But it is my feeling that a considerable amount of the pupil's time could be saved by taking first things first.

There seems to be a group of teachers who feel that a social arithmetic makes the mathematical processes more meaningful. There is reason to question this stand. The contribution of the social uses of arithmetic to the understanding of the processes has not been investigated, to my knowledge. Until that time it may be well to develop a rule of thumb procedure—that of including the social uses of arithmetic solely for the purpose of motivation.

VI. Miscellaneous Studies. I cannot close this paper without mentioning two research studies that have appeared in the very recent issues of the *Elementary School Journal*. These studies, it seems to me, are getting at something fundamental. The first study is reported by Gertrude Hendrix, and is called, "A New Clue to Transfer of Training."[12] The purpose of this study was to determine, in the words of the author, "to what extent, if any, does the way in which one learns a generalization effect the probability of his recognizing a chance to use it?" Three methods of instruction were compared. The first, a generalization, was stated, and then taught. The second method was called the "Unverbalized awareness procedure." In this method, the stage was set so that the subject had a chance to apply the generalization as soon as it "dawns on him." The third method was called the "conscious generalization method" in which the subjects were led to state the generalization as soon as discovered. The interesting part of this study is that the "unverbalized awareness procedure" produced the best results. The subjects used in the experiment were adults. Would the same results follow with children of the elementary school age? This study should be taken seriously by those teachers who advocate teaching generalization. What about those neatly boxed in generalizations so often found on blackboards and in the textbook?

Another interesting study is reported by Ned A. Flanders under the title of "Verbalization and Learning in the Classroom."[13] Extensive records were kept of classroom procedures by means of recording devices. The study attempted to show that there is a relationship between the language of communication and the language of thinking. Among other things the study reports a significant correlation between the kinds of statements made about per cent and the student's individual learning status in that area. This study is approaching a vital problem. If we are to discover anything fundamental in the learning process about the way children learn arithmetic we certainly must find out more about what and how a child thinks. For the classroom teacher this study would seem to indicate that an ability to express ideas verbally is an index of the child's ability to use the idea in a logical pattern. For you and I as teachers it would seem that we should encourage the child to express quantitative ideas in order to get a measure of the extent to which he can think quantitatively.

[12] Hendrix, Gertrude. "A New Clue to Transfer-of-Training." *Elementary School Journal*, 48: 554, June 1947.

[13] Flanders, Ned Allen. "Verbalization and Learning in the Classroom." *Elementary School Journal*, 48: 385, March 1948.

In summary, I wish to call your attention to the fact that in spite of the gaps in research covering the elementary field there are a number of studies which definitely point out a direction in which it is safe to go until further evidence is obtained. True, much of this research has been done on the first to third grade level. Yet, by extrapolating a little beyond date these researches give some indication of what can be expected on the 6th and 7th and 8th grade level. The alert teacher will make cautious use of such extrapolation for the purpose of improving her instructional program.

The National Council of Teachers of Mathematics Membership by States

	Jan. 1947	Jan. 1948	May 1948
Alabama	79	77	86
Arizona	17	26	28
Arkansas	64	58	61
California	264	291	311
Colorado	80	75	81
Connecticut	119	114	120
Delaware	23	24	24
D. C.	105	132	131
Florida	102	130	136
Georgia	40	67	77
Idaho	4	7	6
Illinois	475	473	497
Indiana	137	318	349
Iowa	129	107	122
Kansas	166	150	150
Kentucky	69	68	80
Louisiana	80	100	98
Maine	34	28	31
Maryland	112	114	124
Massachusetts	229	238	249
Michigan	241	213	226
Minnesota	138	144	163
Mississippi	34	42	43
Missouri	133	122	123
Montana	20	24	24
Nebraska	79	78	91
Nevada	4	10	8
New Hampshire	21	31	32
New Jersey	277	312	336
New Mexico	18	42	35
New York	513	643	660
North Carolina	103	96	140
North Dakota	19	15	21
Ohio	348	331	358
Oklahoma	97	81	103
Oregon	77	80	74
Pennsylvania	412	438	463
Rhode Island	37	36	39
South Carolina	64	86	86
South Dakota	21	25	26
Tennessee	88	97	99
Texas	200	208	228
Utah	13	10	13
Vermont	16	27	27
Virginia	114	123	141
Washington	76	67	70
West Virginia	43	52	54
Wisconsin	179	197	203
Wyoming	19	26	26

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Name	Address

Algebra Problems and Problems in Life*

By PHILIP PEAK
University School, Indiana University

THESIS I of the Second Report of the Commission on Post-War Planning[1] says, "The school should guarantee functional competence in mathematics to all who can possibly achieve it." Then follows that very useful check list, composed of 28 items, as an aid in determining whether the student has achieved functional competence. There has been a great deal of controversy over the interpretation of the two key words, "functional" and "competence." This is to be expected, since both words are variables and, if one supplements the other, the range may be great. In my opinion, our mathematics, to be functional, must be adequate to fulfill our needs both now and in the future. This means that each individual will place a different demand on methametics.

Now to be competent in mathematics we must be able to use all we need in an efficient manner. Since efficiency is never one hundred per cent, this becomes a relative matter and allows for wide variations. Therefore, we may have competence in the group while the individual competence ranges from what we often consider none to a very high degree. This presents the question of prime difficulty, namely, "Is there a way we can evaluate and determine this functional competence for each individual?"

For several years I have asked my beginning methods students to put down their interpretation of functional competence, but seldom do they go beyond the formal abilities of number manipulation, formulae, etc. The fallacy of this approach is the isolated character of mere number activities.

What is mathematics? Thousands have given answers without too much success. Perhaps the reason for this is the philosophy and broadness of the subject. Or it may be the pattern of thought which is embodied in mathematics.

It is my purpose today to show you how this pattern of thought is similar to that used in all thinking. Can the sociologist, the economist, the theologian, the educator, and all the others use this same pattern of thought in solving their problems? Thinking in all of its forms involves a problem; therefore, it involves mathematics. Why not use the mathematical mode of analysis on all our thinking? Many times we do not know what the solution will be. Many times we do not need the explicit solution—merely information on general behavior.

This is certainly true in our mathematics problem. Consider, for example, the area of a rectangle whose dimensions measured to the nearest inch are 54″ by 23″. These cannot be exact; and even though we perform a perfectly rigorous operation, the result is still approximate. It will vary from 1,203 square inches to 1,273 square inches; and, with the data involved, this is the best we can do. Mathematics is a precise method of thought. If we feed into it precise facts, we get precise results; if we feed in approximates, out will come approximations. The mode of thought is not affected by the data.

Mathematics as a method has been handicapped by those over-zealous disciples who believed in its power but also believed that anything mathematical could be reduced to any degree of precision desired. Perhaps the most common illustration of this is the fallacy of assigning

* Presented before the High School Section at the annual meeting of The National Council of Teachers of Mathematics at Indianapolis in April, 1948.

[1] The Second Report of the Commission on Post-War Plans, THE MATHEMATICS TEACHER, May, 1945.

number values to letter grades and then using that sacred "grade point average" to determine class standing. Were the original letter grades precise to four significant digits? Mathematics does not perform miracles; it is only an honest worker. Then why not use it as a set of guide posts to aid us in solving the problems of life?

This does not mean that all life's problems will be reduced to numbers. Not all mathematics problems are reduced to numbers. Many of you have studied mathematics courses in which very few numbers were used. But whether you use numbers, letters, or the hieroglyphics of Timbuctu, you organize your thinking in the same way. Since this is the case, why not use as one criterion of functional competence, the ability to use mathematical methods in the solution of life's problems? This must be taught; the method will not transfer automatically. It cannot be tested by checking answers, for many of life's problems have no single answer. To check we must evaluate the analysis rather than just the result.

With this philosophy in mind, I should like to present a typical situation from a high school algebra class. We do this in an attempt to lead the student in transfering his ability to think in mathematics to that of everyday life. This involves verbal problems or story problems which we use in a broad sense. We will take one problem and read into it many possible conditions. What is the probable setting; what came before; what would happen if this word was changed; etc.? Our purpose is to get the pupil to feel that the problem arose from a specific need and that its solution will probably lead to another need. This is the chain reaction of life, and our book problems should be studied in the same way.

The student may then develop an insight into implied information and hidden relationships. True, we must use our imagination, but we also do this in life. We are never aware of all the conditions we may have to face if we choose a particular career, buy a house, or even teach an algebra class. We merely analyze the problem on the basis of our foresight and then draw our conclusions.

It is difficult to teach pupils in such a way that they will reduce all life's problems to the mathematical method. When our students meet verbal problems, they usually feel no need for solving them. They say they never mix 30-cent coffee and 50-cent coffee, or fill a tank using three pipes flowing in and two pipes flowing out. When we ask them if they ever solve problems except in school, the answer is usually, "No."

With this introduction we are ready to vitalize problem solving. We do this by asking such questions as: Did you go to the show last night, or who saw last night's ball game? From these questions we get sufficient material for many problems. We analyze these to determine what the problem was in each case. If we take the case of a basketball game, we have an illustration of many possible solutions and no way of checking which is correct.

But let us take a simpler illustration. Joe asked Becky to attend a show with him tonight. What is the problem? Is there any implied information not mentioned in the statement of the question? Some of it might be: there is a show; it will take money; Joe has the money; they have a way of getting there; Becky's parents will let her go; and much more. Now any part of this implied information may pose a problem in itself. Let's examine the item of permission from Becky's parents. The factors which effect its solution are: today is washday and Mother is in a poor humor; Becky has been out her quota of nights this week; Becky's grades haven't been good and she should stay home to study. The solution to one or all of these may be needed to solve the original. Becky proceeds on the basis of past experience; once she hurried home and washed the dishes; once she cared for the baby; once she got Dad to work on Mother; another time she had her brothers offer

to help. She chooses from this list the item or combination of items which she feels will most likely be successful under present conditions. She will not know the answer, but she must assume one because she must accept or reject Joe's invitation immediately. Therefore, she assumes the answer correct, accepts the invitation; but that brings out the problem of what to wear. This too must be based on implicit information: where will Joe take her—just to a show, to a swanky restaurant, a jive joint, or a drive-in? Here again the problem must be solved on the basis of past experience, and the answer assumed correct. This must be thought over carefully because the problem can never be worked over. If Becky finds she has misjudged her mother and must break her date, she may lose Joe; and if she wears the wrong dress, she may be embarrassed. The analysis makes a great difference here.

By this time our students have begun to realize that the solution of problems is not unique in mathematics; that success depends on the proper analysis; that every problem uses past experiences and the better these experiences are used, the more likelihood of success. We emphasize here that the major purpose of verbal problems is to give the pupil experience in using the mathematical method and the solution of any particular problem in itself has little value.

We are now ready to take a problem from the text and make our comparisons. The students select the problem since any one will do. Suppose they chose: "A freight traveling 30 miles per hour is followed two hours later by a passenger traveling 50 miles per hour. In how many hours will the passenger overtake the freight?" How does the implied information compare to that in Becky's problem? Here we must make assumptions as to the relationship of distance, rate, time, the number of stops, the route traveled and others before we can solve the problem. Suppose we do all the above; we still have some choice in the equation to use and the steps used in getting the answer from it. Our choice, as before, depends on past experience.

The students feel that text problems are easier because they say we put in just what we want, while in life, things cannot be so well controlled. Problems become more than a task to be performed today and forgotten tomorrow; they become an aid to the everyday thinking of the pupil.

In conclusion let me make some direct comparisons:

1. In verbal problems, we must read between the lines just as we must imagine conditions in life.
2. In verbal problems, we must recognize the real problem which is often hidden; this is also true in life. If you and I could recognize our real problems, the need for psychiatrists would be lessened.
3. In verbal problems, we must analyze the best approach from the many possible; this is more true in life, since time does not turn back.
4. In verbal problems, we must recognize those parts that have no bearing on the problem and not waste our time on them. In life this is harder to do, but we spend a great deal of our time doing things which are of no values to us or to others.
5. In verbal problems, we must anticipate and estimate what the possible solutions may be, and certainly this is true in life.

In the light of these five items, our teaching of mathematics requires less time with the pencil and more time in deep thought. Mathematics is thinking. Manipulation of numbers is only an aid in setting down those thoughts clearly and concisely. Mathematics is beneficial because its symbolism enables one to isolate attitudes and facts. By mathematics, we can draw a fine line between logical analysis and personal attitudes. The things we work with in mathematics are not of a controversial nature; therefore, the steps in problem solving can be set out and we can see just what goes on at every stage. There is no confusion of logic and emotion. However, the pupil may be an excellent thinker in mathematics and it never occur to him that the same kind of thinking is used in everyday affairs. At every opportunity we must compare the problems of

mathematics to those of life. For real understanding, these life problems must be those the students meet daily. Analysis must go deep into the problem. Every person in the world faces problems day after day, and solutions of these are only obtained by isolating the real obstacle, making basic assumptions, accepting physical facts, and then drawing conclusions on the basis of our experiences. This is problem solving in mathematics or in life.

Teachers of Mathematics*

By Howard F. Fehr
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There are persons who do not like to teach who are conducting classes in our schools. There are persons who do not like mathematics who have been assigned to handle algebra classes and to instruct in arithmetic. There are men and women who do not understand mathematics yet they are engaged by boards of education to instruct in the subject. There are people with no sympathy for, nor understanding of children's desires and ambitions who daily browbeat their assigned pupils into fearful obedience through routine lessons. There are pedants who asperse educational theory and methods of learning, seeking only to perpetuate in their pupils the memorized meaningless manipulation of symbols that constitutes their entire stock of knowledge. They are blind to the world about them. These are not the teachers of mathematics. But we are teachers of mathematics if we have the following attributes.

We are teachers of mathematics first of all because we like to teach. We enjoy telling facts, giving knowledge to others, whether they be youths or adults. We like to develop within others the spirit of inquiry, the search for truth, and in so far as we know how, a mind that becomes an instrument for the understanding of life. How it came to be that we prefer teaching to any other career we do not profess to know. It may have been innate, or the effect of chance events upon our life, or some other peculiarity of circumstances, but this we do know—we prefer to teach rather than to do anything else in our life.

Not only do we like to teach, but we also like mathematics. The subject in all its phases, to whatever degree we have studied it has given and continues to give us real satisfaction. We like the challenge it gives to correct and rigorous thinking, we thrill at its beautiful proofs, and we never tire of it as year after year we unfold the same but ever new subject to our pupils. The problem sections in the current periodicals are proof of this enjoyment. We like other subjects and other knowledge, but we like mathematics most of all.

Because we like to teach and also like mathematics it is natural that we like to teach mathematics. We can teach it because we *know* mathematics. This does not mean that we know all the mathematics nor that all of us know the same mathematics. Each of us during his lifetime has created for himself a foundation of mathematical knowledge upon which he stands as an authority as he teaches his classes. While this foundation must be bed rock, it is not the same size for all teachers. The teacher of arithmetic in the elementary school needs to know not so much as the high school teacher of advanced algebra who in turn needs to know not so much as the college teacher of differential equations. How much mathematics do we know? We know sufficient to meet our every need that arises in the teaching we do.

As teachers of mathematics we are continual students of mathematics. We subscribe to the age-old dictum: *"Thou that teacheth, teach thyself."* This does not mean that we necessarily study more advanced mathematics. We study the subjects we are teaching for there is ever something new to be learned in the most elementary parts of mathematics. We study new phases and new developments that are taking place in applied mathematics. We study more of the foundation on which the subject is built. For pure delight we

* Paper read at the annual meeting of The National Council of Teachers of Mathematics at Indianapolis in April 1948.

may study advanced graduate subjects in pure mathematics. Whatever it may be, we teachers of mathematics are day by day students of our subject and we learn much that is new every year of our career. We are the kind of scholars in a scientific world that Professor MacDuffee pleaded for in his retiring address as president of the Mathematical Association of America.

Continual students of mathematics are well informed on the history and applications of mathematics. We have learned how and why mathematics came to be the great branch of human knowledge that it is, and we reflect this knowledge in our everyday teaching. We know the dependence on mathematics of so many other branches of knowledge for expressing their fundamental concepts and relations. We know applications of mathematics in sound, electricity, light, mechanics, dynamics, geography, astronomy, navigation finance, economics, medicine, art, and even relativity and nuclear physics, to a sufficient degree to show our pupils how mathematics explains the universe round about them and the continued need and growth of mathematics for such purposes. The teachers of mathematics know mathematics.

They use this knowledge to make mathematics interesting to their pupils. They know full well that all learning is finally a product of individual initiative and not of superimposed skills and facts. The teacher's main objective is to make pupils want to learn and pupils usually learn that which they like. So we make our subject meaningful, alive, thrilling, satisfying, and always within the comprehension of the pupil. We make our pupils like mathematics; at least we try very hard to do this. We do this because the supreme obligation of mathematics teachers is to *humanize* mathematical knowledge, to interpret the creations and discoveries of the pure scientists so that youth may understand and appreciate them, and use them.

We teach mathematics in an interesting and captivating manner also because we like human beings, especially children. We respect and honor the individual dignity of each personality in our classes. We tolerate their peculiarities and sympathize with their difficulties. We hold dear and precious the mind of each of these whom we sometimes refer to as little brats because they can be mischievous, troublesome, and trying. But we recognize in each one of them an important future citizen of democratic America. Knowing the value of mathematical training for everyday living in our country, small though it may be for the many, and large as it is for a few, we have a responsibility. We guarantee to secure for these children the necessary mathematical equipment for the proper development of their characters and their careers.

A child desirous of learning to play the piano came to a music teacher. "We shall make of you a great musician," said the teacher, "so we shall start with the proper foundation in technique. You must learn to play scales, chords, and arpeggios for they are the foundation of all music." At the end of the year the child had made much progress and he was passed on to a teacher of more advanced pupils. The new teacher said, "we shall improve your ability to play scales, chords, and arpeggios, and you shall go on to grand arpeggios and scales in four octaves." At the end of that year the child was most adept at playing scales, chords, and arpeggios, and again he was passed on to a teacher of more advanced pupils. This went on from year to year. At the end of ten years the child had become the world's best scale and arpeggio player but no one would come to hear him perform, for who wants to hear just scales and arpeggios? Nor had he learned to appreciate how to apply these scales, chords, and arpeggios, through the laws and rules of harmony to the creation of a Bach fugue, a Beethoven sonata, a Brahm's symphony, or a Mendelsohn concerto. To him music had become a clever but meaningless dull repe-

tition of skills and techniques in scales, chords, and arpeggios.

Mathematics has its scales, chords, and arpeggios also. They are the fundamental operations of arithmetic and algebra, special products, factoring, radicals, and grand superimposed radicals, simple fractions, and fractions in four octaves, postulates, definitions, theorems, functions, derivatives, and integrals. We could develop these skills year after year until our pupils could perform them accurately and correctly with speed. But the teachers of mathematics are aware of the folly of narrow and meaningless instruction. Once a few skills and their meanings have been learned, they teach their pupils to play little tunes in arithmetic and algebra in the derivation of a formula from a table of values or in finding the cost of painting the cellar floor. As progress is made in developing new and more complex concepts capable students are taught to compose and perform algebraic sonantas and geometric concertos, and the less capable are taught to recognize and appreciate the higher mathematical creations. The development of the formula

$$i=\frac{24\,I}{P(n+1)}$$

for determining the rate of interest on monthly installment purchases is a performance of a little symphony in algebraic thinking. It gives real satisfaction to the few pupils who perform its derivation and also a sense of pleasure to the many pupils who follow the derivation and recognize its potentialities when applied to everyday economic affairs. The plotting of the radius of action of an airplane patrol by the use of plane vectors is a concerto in geometric design with variations of the harmonic mean. The use of the Beta and Gamma Functions and probability in n-dimensional space in determining the reliability of statistics in sampling problems is a grand mathematical symphony. The teachers of mathematics develop composers, performers, and an appreciative and understanding audience in the harmony of mathematics according to the needs, skills, capacities, and interests of the pupils.

The teachers of mathematics can do this because they know how to teach. We recognize that what we like to do needs careful and critical study so that we may do it correctly and efficiently. Just as the boy who likes to run studies under the care of a coach so that he may learn correct body movements and correct breathing and eventually win the race so we have studied under capable educators who have proved their ability to develop successful teachers. We know and apply certain fundamental theories of learning, of motivating learning, and of developing mental processes. We are continual students of pedagogy. We study our own teaching procedures day by day that we may discard those devices that prove of no value and that we may maintain and improve successful techniques. Year after year we improve in the presentation of our subject. We also believe that in our profession, as in any other, there are basic laws but that as new knowledge is gained it is frequently necessary to abandon or alter these basic laws so that we may truly progress in the Art of Teaching.

Finally, the teachers of mathematics are what Richard R. Werry* has termed "outminded." "Outmindedness is looking through one's immediate subject into other subjects, in a constant awareness that one's own knowledge is only a small tower from the top of which one may survey ever larger portions of the vistas of human knowledge. The higher a teacher can build his own tower, the more distant the horizons he can perceive and refract through the lens of himself to his students But he must always stand on top of the monument looking out from it. He must never permit himself to be moated inside."

* Richard R. Werry. "With the Tongues not of Angels but of Men." *Bulletin of the American Association of University Professors*, Autumn, 1947.

Thus we are persons who specialize in our subject mathematics but always generalize in our own interpretations and in our teaching. We merely use the subject we know and like, best as a means of interpreting the universe and our environment to our pupils. Other subject matter teachers do the same. Then a pupil begins to see human knowledge from many points of view, but always as an integrated body of knowledge. We make the pupil aware of and equipped in mathematical modes and processes that form a vital part of investigation of the whole structure of knowledge.

In summary, we are teachers of mathematics because we like to teach, we like mathematics, we know mathematics, and we continue to study mathematics. We know how mathematics came into existence and grew to be a great branch of human knowledge indispensible in modern life. We know how to teach, how to make mathematics, interesting, and how to motivate pupils to its study. We develop meanings and appreciations. We like children, respect their difficulties, and develop those teaching methods that prove most effective.

Each one of us has built his house of mathematical knowledge, some small, others large, but we are all engaged in rearing our house to greater heights. We stand on top, master of our house, viewing the large and ever growing panorama of human knowledge that stretches out before us. To the best of our ability we interpret this knowledge to the pupils who come to our house. Teaching in this manner we make an important contribution to the totality of knowledge and thought, to the progress of civilization, and best of all, to human happiness. These are the teachers of mathematics.

The Place of Mathematics in General Education*

By Edward A. Cameron
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The place of mathematics in general education was discussed at least as long ago as some 2500 years, when the Pythagoreans established the quadrivium of arithmetic, geometry, astronomy, and music, subjects which were to be considered the heart of a liberal education for many centuries. That the subject is still being discussed today can be readily verified by consulting almost any recent issue of The Mathematics Teacher. The *Eleventh* and *Fifteenth Yearbooks* of the National Council of Teachers of Mathematics contain much valuable information on the subject under discussion, and I heartily recommend them to any teacher of mathematics who has not yet read them.

I do not propose to review the many discussions and arguments which have been made about the role of mathematics in education. Nor do I, on the other hand, pretend that my remarks will be characterized by any great originality. I would like simply to discuss briefly some of my own views, and raise some questions which seem to me to be particularly pertinent today.

Let us recognize at the outset that there is a great difference between the educational potentials of a subject and the realization of those potentials. Euclidean geometry may have the latent possibility of instilling in a student an understanding and appreciation of logical reasoning—as it is said to have done for Abraham Lincoln—but if it is taught in such a way that students merely memorize proofs in order to parrot them back without understanding the reasoning involved, this latent possibility will never be realized. Whether a subject contributes appreciably to a student's education frequently boils down to whether the subject is taught properly, and whether the student has the intellectual equipment to assimilate it. If there is one thing which has impressed itself upon me more and more with experience, it is the existence of individual differences and their importance in our educational system. More of this later.

You have heard, I am sure, appraisals of the educational value of mathematics ranging from those of some of the so-called progressive educationists, who would delete from the curriculum all mathematics beyond that needed in buying groceries, to those people who hail mathematics as the touchstone which will enable its possessor to save the world. It is trite but true to say that neither of these extremes is correct nor even very sensible. Instead of trying to make any such blanket appraisal let us rather ask the concrete question: What mathematics should be taught to whom, and how? I shall not presume to answer this question, not because I do not have time, but because I do not know the complete answer. I do not believe that anyone does. But that must not keep us from trying to find it, or at least to get closer approximations for it. Properly speaking, of course, there is no one correct final answer but many, because human society is not static and human knowledge is not static, and what is best suited for one time and place is not for another.

But to get back to the claims of mathematics for a place in our educational program, there is a minimum amount which every normal person needs to know, and consequently should be taught in our public schools. All but the most radical would probably be able to reach essential agreement on such a minimum, which would include arithmetic, graphs, simple

* An address delivered before the Mathematics Division of the South Carolina Education Association, March 18, 1948.

algebraic formulas, some mensuration and informal geometry, etc. Most of these topics would be taught in the first seven or eight grades or at least by the middle of the first year of high school. It is when we attack the problem beyond this minimum that great differences of opinion really arise. You are all familiar with the debatable questions: how much mathematics should be required of students intending to go to college, and how much of students not intending to go; should the mathematics given be algebra and geometry, taught separately or organized into so-called general courses; during what years should the various topics be taught; how logically rigorous should the treatment be; and so on. Now these are vitally important questions and worthy of the best thoughts that our educators and mathematics teachers can give them. They should not be answered on a basis of prejudice nor from a background of ignorance. I am afraid that many times wrong answers have been given because many people competent in mathematics have not had the interest to study the whole educational picture, and many people interested in education have not had the proper knowledge of mathematics. More cooperation and better understanding between mathematicians and educators would help greatly in producing better educational programs.

As I intimated previously, I am convinced that the schools must begin to take more account of individual differences if they are to perform their proper function in our democratic society. Democracy does not mean that everyone should receive the same education. To me it means that everyone should receive the type of education which will enable him to develop his innate talents to the utmost, and this includes the most gifted as well as the subnormal. In my opinion, and in the opinion of others who have given some serious thought to this problem, it is the student of superior ability who is being shamefully neglected in many of our public schools—and colleges too for that matter. In our effort to care for the great masses we have diluted our courses and talked down to the students in such a way as to tend to stifle intellectual development in those we need so desperately to furnish the leadership in science, industry, government, and other phases of human activity. Different programs and different classes at the high school level for students of markedly different ability is the only way I can conceive for effectively taking care of these individual differences. This proposal immediately raises at least two questions. One is budgetary: how can smaller schools, especially, hope to have a two-track or three-track program? The other is: how can superior students be selected from the masses? The first question is a difficult one, and involves considerations beyond the scope of my experience. Concerning the second I do have some convictions.

While I have never had any experience with tests at the high school level, I have had considerable to do with placement tests in mathematics at the University of North Carolina. We have used them for about ten years for sectionizing students according to ability in mathematics, and have found them to be very effective—not 100% but still plenty good enough for practical segregation of the students into three groups. This program of separating the students into sections according to their various capacities and designing courses to fit their needs has been generally successful for us. I am convinced that work along this line would be worthwhile at other educational levels. It is an interesting sidelight, that students selected solely on the basis of superior grades on a 40 minute mathematics test do distinctly superior work in all other subjects as well.

I do not have the time to pursue this subject further, but I can not reiterate too strongly my firm conviction that more study of the problem of individual differences, and particularly of means for

enabling gifted students to avoid the wasteful and stultifying treadmill pace of the mediocre offers great hope for improvement in our system of universal education.

The question of what kind of mathematics should be taught in high school is in my opinion closely connected with the problem of individual differences which we have just discussed. For many students the ability to understand and profit by the study of mathematics beyond the minimum topics mentioned earlier is definitely limited. To attempt to give these students mathematics of a degree of abstractness beyond their capacities serves no useful purpose. I hasten to add, however, that we must be careful to ascertain that a student really does not have the capacity for learning mathematics before advising him to discontinue its study. His poor performance may be due to poor teaching or just plain laziness on his part. In passing I might say that for most students incapable of learning high school mathematics, I seriously doubt the value of a formal college education.

Students who can learn mathematics beyond the minimum mentioned earlier—and there are more of them than many people think—should be given mathematics up to the limit of their capacity of understanding. There is no need before an audience of this kind to labor the advantages of a good foundation in high school mathematics for students capable of assimilating the algebra and geometry which constitute its chief content. The fields of science, engineering, statistics, etc. which require mathematics as tools immediately come to mind. Less immediately apparent but pertinent for more people is the training in preciseness of statement and habits of orderly thinking which should be emphasized in teaching mathematics. An introduction to logical reasoning based on explicitly stated postulates is afforded in demonstrative geometry, and very few other places in our curriculum. I could go on, but instead I want to recall what I said before, that these benefits do not automatically accrue to a student just by attending a course in mathematics. The student must be intellectually capable of assimilating the subject, he must do a reasonable amount of work, and the subject must be properly taught.

On the question of sequential courses in algebra and geometry versus general mathematics in high school, I am not competent to pass judgement. I have taught both types of courses in college and have found advantages in both. A general course can be excellent and contain good sound mathematics, or it can be a collection of frothy trivialities, capable of intellectual stimulation to no student of any appreciable ability. Throwing a lot of different topics together in a book does not automatically make that book broadening and a fit instrument for educational purposes. It takes work and thought on the part of both author and teacher to make a general course really good. I am not opposed to general courses as such. I am merely pointing out some of the travesties committed under the title.

With regard to the method of presenting mathematics, the intuitive, inductive approach versus the formal, deductive approach perhaps is no longer considered a debatable question in educational circles. The child, like the human race, learns by proceeding from the particular to the general, from the concrete to the abstract. This psychological principle can not be ignored. However, if the student never gets beyond the particular and the concrete, if he never learns at least in a limited way to generalize and abstract, then he has not really learned what mathematics is all about. When and how to start this generalization and abstraction can not be decided by a man sitting at a desk writing a speech. It can only be decided where so many important questions should be decided; namely, on the basis of actual classroom experience.

I would like to emphasize this last

point. It has been my experience that it is virtually impossible to determine without actual trial in the classroom whether a particular book or a particular method of presentation is going to be successful. Armchair strategy is a most undependable thing in the business of teaching. Preliminary planning must be done of course, but do not stick to a plan if it does not work in your particular situation. Personalities differ and a method that works for one teacher will not work for another, and a scheme which is effective in one class may be a flop in another. This is one of the things which makes teaching interesting and continually a challenge. No two classes are exactly the same, and a teacher at any time may have to improvise to meet a situation never hitherto encountered.

In the formulation of educational policies I think the classroom teacher should have a greater voice than is true at present in many of our schools. Very unfortunate, I think, is the idea that because administrative jobs carry larger salaries then these jobs are more important than teaching. No job in a school is as important as teaching. Teaching is the only reason for the existence of a school, and the position of a teacher should be one of dignity and honor second to none. The average teacher in this country is poorly paid in comparison with other professions, but that should not be taken as a measure of the importance of the position. And I repeat, the opinions of teachers in educational matters should carry more weight than they now do in many of our schools.

In closing there is one thing which is perhaps more important in mathematical education than anything I have yet mentioned. That is the personality of the teacher and his or her enthusiasm for the subject. Some teachers have the wonderful gift of being able to invoke a measure of inspiration in their students that is of far greater worth than all the methods and plans that could be devised in the next hundred years. If we had enough mathematics teachers with this divine spark, no one would question the place of mathematics in general education.

◆ AIDS TO TEACHING ◆

By

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BOOKLETS

B. 9—*Overweight and Underweight*

Metropolitan Life Insurance Company; Madison Square, New York, N. Y.
Booklet; $5\frac{1}{2}'' \times 7\frac{1}{2}''$, 32 pages; Free.

Description: This booklet showing a mathematical aspect of health contains fifteen pages of general description of dangers of being over- or under-weight, and ideas on reducing; thirteen pages of calorie equivalents, and then miscellaneous material on exercises, etc.

Appraisal: Why not have pupils use real material from outside the textbook to make graphs such as are contained in this booklet in the junior high school? Having children figure the calorie content of their meals and then keep a running graph for a month or so would give them practice in looking up figures in tables, using fractional parts of calories listed, making and interpreting graphs, in addition to showing relationships between their diet, their weight and their general health.

B. 10—*You Bet Your Life*

Travelers Insurance Companies; Hartford, Connecticut
Booklet; $6'' \times 9''$, 32 pages; Free

Description: There are fifteen full-page cartoons explaining common practices which lead to automobile accidents; also fifteen pairs of tables contrasting accident statistics for 1941 and 1946. Paragraphs accompanying each pair of tables help to interpret their meanings.

Appraisal: This booklet provides a great deal of material for construction of graphs, especially to compare two sets of similar statistics for different years. Even more important, it suggests many similar surveys which mathematics classes could make of accidents in their own neighborhoods or cities. This would lead to methods of collecting, recording and interpreting data and would give opportunities for drill in addition and percentage which is always needed but often avoided because it is boring to both the teacher and the pupils.

B. 11—*A Career in Life Insurance Sales and Service*

Institute of Life Insurance; 60 East 42 Street; New York, N. Y.
Booklet; $5\frac{1}{4} \times 8\frac{1}{4}''$, 16 pages; Free

Description: This book is provided as vocational guidance for those interested in life insurance work. It tells the types of jobs available in sales and service, the qualifications needed, the training required, the usual incomes and chances for advancement.

Appraisal: It may be more useful to have such vocational booklets scattered throughout the school where they will be likely to reach the pupils most interested in them rather than concentrate them in a guidance office where pupils seldom go. Those interested in mathematics and figures tend to seek jobs using them and often consider insurance such a field. This booklet stresses the sales and service positions, but shows that even there some knowledge of mathematics makes the work easier.

CHARTS

C. 4—*International Metric System*

Superintendent of Documents; Government Printing Office; Washington, D. C.
Chart; 28.1″×44.1″; (Catalog No. C 13.10: 3/7); $.40

Description: This chart printed in black and yellow illustrates and compares the English and Metric systems of measurement for length, area, volume, liquid and dry measure. Since all illustrations are lifesize they are the best substitutes for measuring sticks, areas and volumes themselves. Measures to be compared are placed side by side to facilitate comparison.

Appraisal: The chart is well labeled and clearly printed. The paper is very heavy, but not enough so that it could be handled, rolled and unrolled, by young hands. To make it more permanent it should be framed, mounted on heavy cardboard or mounted on the wall permanently. It would be desirable to have an additional chart mounted on heavy board which could then be cut around the outlines of the various units so that they could be used to measure objects around the room and to lay on top of each other for easier comparison. This chart should be useful throughout the elementary and junior high school years.

EQUIPMENT

E. 5—*The Circle Club Kit*

Mary Hughes, Hughes Educational Kits; 1640 Connecticut Avenue, N. W.; Washington, D. C.
Arithmetic Teaching Kit, $25.

Description: The Circle Club Kit is made up of 31 cards, 17″ by 19″, a 35 mm. film-strip of 36 frames, a teacher's manual and a class progress chart. On the face of the cards are printed, in bright-colored circles, the 31 answers from the 90 multiplication and division facts; on the reverse side of the cards, in similar circles, are the related facts with answers. The film-strip covers the same facts, in sequence of products, as those on the cards. The teacher's manual illustrates a method of developing the facts with small children, makes suggestions for using the cards and film-strip in drilling, and gives samples of tests which might be used for checking. The progress chart is arranged to show, by some convenient symbol, when each child has mastered each of the cards.

The kit is intended for use with a method of developing the multiplication and division facts which is illustrated in the accompanying manual. The facts are developed, not through the use of these aids, but by using objects in the pupils' surroundings which may be grouped and combined to illustrate each fact. It is described as "visual, tactual, auditory," and by using this method children can get a better concept of the meaning of multiplication and division. They may be led to see more plainly that numbers are made up of factors; this concept should prove helpful in later work in long division and in fractions. The one large number on the face of the cards may allow rather free play for the minds of some youngsters who may wish to associate with it certain addition or subtraction facts, or even fractional relations. This might be encouraged with older children in review drill work, but would confuse many of the younger ones so that the teacher would have to be very careful to keep before the children the idea of multiplication and division within the given range until they have mastered the desired facts.

Appraisal: Since the cards are large, and take up considerable space placed side by side, they would have to be used a few at a time; they could hardly be used by small youngsters for individual study on certain varied facts. Being large, they are awkward to manipulate; furthermore they are not very substantial and would not stand up well under constant handling. They need to be bound in some firm edging to protect them against tearing and breaking. The figures are small but clear for

ordinary classroom distances. The cards are colorful. The film-strip provides a means of stimulating and speeding drill work. The chart as an indicator of pupil progress is always a satisfactory device with young children.

The cost of the kit puts it beyond the reach of many teachers who might use it. However, the writer feels that the method suggested could very well be carried out by any teacher without these particular aids; probably many teachers have used similar devices of their own making. (Reviewed by Miss Cecelia C. Callanan, Wellesley Hills, Massachusetts.)

FILMS

F. 16—*The Circle*
F. 17—*Chords and Tangents of Circles*
F. 18—*Angles and Arcs in Circles*
Knowledge Builders, 625 Madison Ave, New York, N. Y.
16 mm. sound film; each title 1 reel; black and white. $40. 1944

Content of F. 16: In this film the circle is introduced by showing its everyday use in wheels, highway cloverleafs, and city planning. The film defines and illustrates the following terms connected with circles: radius, secant, diameter, tangent, major and minor arcs, semi-circle, central angle, and chord. Theorems on the equality of central angles, intercepted arcs and chords in the same circle or equal circles are stated and proved.

Content of F. 17: Additional applications of the circle in city planning and architecture are shown in this film. The theorem, "The diameter perpendicular to a chord bisects the chord and its arcs," is stated and its proof demonstrated. Other theorems regarding the relation between lengths of chords and distances to the center of the circle or equal circles are stated without proof. The second part of this film defines and illustrates tangents, the construction of tangents, the relations between a tangent and a radius drawn to the point of tangency, common external tangents and common internal tangents. The theorem, "Tangents to a circle from an outside point are equal," is stated and proved. Circles tangent externally, internally and concentric circles are illustrated by moving two unequal circles together until their centers coincide.

Content of F. 18: This film covers the fundamental theorems about the measurement of angles related to circles, namely, central angles, inscribed angles, angles between intersecting chords, angles between secants, angles between a secant and a tangent, angles between a chord and a tangent and angles between tangents. The theorems on the measurement of an inscribed angle and the measurement of the angles between intersecting chords are proved. Each theorem is followed by at least one numerical example. Usually the arcs are labelled so the length of arc can be read immediately.

Appraisal of F. 16, F. 17, and F. 18: Except for the illustrations of the everyday uses of circles, these films use animated drawings similar to blackboard drawings. However, the relationships and equalities of lines, angles and arcs can be shown more effectively by the films than by blackboard drawings since by animated drawings, the circles can be shown moving into different positions. The measurement of arcs by superimposing units of arcs similar to those on a protractor simplify the comparison between angles and intercepted arcs. It is unfortunate that the film does not give everyday examples of all the terms defined such as radius or tangent. Good teaching guides for these films are available.

Technical Qualities: Photography: Very good; drawings are very clear. Sound: Excellent. Content: Average.

F. 19—*Measurement*
Coronet Instructional Films; Coronet Building, Chicago 1, Illinois; Collaborator: Harold P. Fawcett.
16 mm. sound film; 1 reel; black and white—$45; color—$90.

Description: This film illustrates and explains by everyday life experiences of a boy the basic types and methods of measurement such as the volume of the refrigerator, the temperature of an ill child, the area of a canvas sail. The seven kinds of measurement treated in the film are: linear, square, cubic, weight, liquid, temperature, time. A day in a boy's life, from the ring of the alarm clock on through the day—at home, on the ballfield, downtown—highlights the importance of measurement to modern living and motivates the study of means and tools of measuring.

Appraisal: This color film selects very appropriate illustrations for teaching the basic types and methods of measurement; for example, the area of a lawn to be mowed, the distance travelled by bicycle, the settlement of an argument during a dart game by measurement and the building of a model airplane. At times the commentary and acting are somewhat unnatural in order to emphasize the relation of measurement to a certain activity but this is probably unavoidable in an educational film that covers as much information in as short a time as this one. Although the film's content is in terms of pupils' activities that could be illustrated and discussed in a classroom without a film, it is likely that this film will add to the interest in and understanding of the kinds and uses of measurement.

Technical Qualities: Photography: Good, few incorrect exposures. Commentary: Appropriate and clear. Content: Excellent Level: Junior high school.

F. 20—*Global Concepts in Maps*
Coronet Instructional Films; Coronet Building, Chicago 1, Illinois
16 mm. sound film; 1 reel, 400 feet, 11 min.; black and white—$45, color—$90.

Content: In this film Dan, a young boy, hears about maps from a voice that comes from nowhere, but answers his questions very well. This device does not upset Dan and very soon it does not bother the audience, either. It shows the globe as the ideal map, and points out why all flat maps are distortions. Cylindrical, conic and perspective projections are explained as attempts to reduce the amount of distortion. The meaning and use of great circle routes are explained on one of the polar projections. Throughout the film Dan is constantly performing experiments with a cut rubber ball, maps on glass globes (with a light inside), and flat pieces of glass on which maps can be drawn.

Appraisal: Next to the method of having all the models and devices used in this film in every classroom for students to handle, this picture is the best way to teach map projections. The choice of material and the speed of development of the topic are excellent. As there is probably too much covered for a beginner the film should be used for review, or various, short parts of it used in class as each part of the topic is introduced. The mathematical theory of maps is one of the topics avoided by both geography and mathematics teachers; the latter often giving as their excuse that it is *solid* geometry! If we can't unify all of mathematics in the secondary school can't we at least break down the barriers within geometry!

Technical Qualities: Photography: Excellent. Commentary: Novel and Appropriate. Content: Accurate and complete. Level: Senior high school or junior college.

FILM STRIPS

FS. 20—*Introduction to Plane Geometry*
Society for Visual Education, Incorporated; 100 East Ohio Street, Chicago, Illinois
Film-strip; 35 mm.; 52 frames; black and white—$2; 1947

Content: This film-strip defines geometry as the study of the relations of lines, points, angles, surfaces, and regularly-shaped solid objects. It begins by showing pictures of different occupations in which geometry is used. It defines plane surface, point, straight line, curved line,

broken line and angles, and illustrates each with a practical example. For instance, the skyline of a large city illustrates a broken line. The strip ends by asking the observer to answer seven questions on the concepts covered by the strip.

FS. 21—*Basic Angles in Experimental Geometry*

Society for Visual Education, Incorporated; 100 East Ohio Street, Chicago, Illinois.

Film-strip; 35 mm.; 58 frames; black and white—$2; 1947

Content: This film-strip shows the unit of measurement of an angle, degree, and how the size of an angle does not depend on the length of the sides. Angles classified according to size (right, acute, straight, obtuse, and reflex) are defined and illustrated by everyday things, such as an airplane propellor to illustrate a straight angle. Angles classified according to position (adjacent and vertical) are also defined and illustrated. A carpenter square, transit, and protractor are used to illustrate the tools of geometry and the occupations using geometry. The strip ends with eight summary questions for the observer to answer.

Appraisal of FS. 20 and FS. 21: These film-strips will furnish the geometry teacher with the kind of material he usually wants, applications of the content of geometry. The everyday examples of geometry are emphasized in the pictures by outlining with a heavy white line the lines or angles involved. It is unfortunate that the pictures of the occupations using geometry give no indication of how or what geometry is used. The questions at the end will assist the teacher in a follow-up discussion of the content of the film.

Technical Qualities of FS. 20 and FS. 21: Photography: Good. Content: Appropriate for beginning a course in plane geometry.

FS. 22—*Measurement and Measuring—Part I*

Jam Handy, 1775 Broadway, New York, N. Y.

Film-strip; 35 mm.; 38 frames; black and white; $2.

Content: This film-strip shows standards of measurement, accuracy of measurement, the use of the steel rule, the divider and the caliper. The metric units of length are compared to the English units, the fractional divisions of an inch compared to decimal divisions. Drawings and pictures of machine parts are used to illustrate tolerance and the methods of accurate measurement.

Appraisal: Although this film-strip was produced for the training of machinists, it is very usable in a mathematics class in a unit on measurement. The emphasis is on the linear measurement in a machine shop but as such it will show a practical application of skill in measurement.

Technical Qualities: Photography: Good. Content: Limited to linear measurement in machine industry.

FS. 23—*Areas by Integration*
FS. 24—*Double Integrals*
FS. 25—*A Triple Integral*

Society for Visual Education, Incorporated; 100 East Ohio Street, Chicago, Illinois; Educational Advisor: Edwin A. Whitman

Film-strips; 35 mm.; FS. 23—36 frames; FS. 24—43 frames; FS. 25—51 frames; black and white; $2 each.

Content: These three film-strips present a series of graphic illustrations to visualize statements similar to "the fundamental problem of the Integral Calculus is that of finding areas bounded by curved lines." The rectangular coordinate system is utilized to illustrate curves, increments, elements of area and of volume. FS. 23 shows how to find an area between $y=\frac{1}{2}(3-x)$ and $y=\frac{1}{2}(x^2-3x)$ by adding rectangles each having a width equal to Δx. The fundamental definition of Integral Calculus, that the limit of a summation is a definite integral is illustrated. FS. 24 shows how the summation of elements of

area, ΔA, can be used to determine area. This summation is then obtained by the evaluation of a double integral. Sample problems for testing the student's understanding are included at the end of the strip. FS. 25 obtains the volume inside the cylinder $x^2+y^2=16$, above the plane $z=0$, and below the cylinder $y^2=16-4z$ by using an element of volume, Δy, to build columns and slices. Variation in order of integration and limits for different curves and surfaces are discussed in all three strips.

Appraisal of FS. 23, FS. 24, and FS. 25: These film-strips make it possible for the calculus instructor to have available accurate drawings of curves to illustrate the computing of areas and volumes by integration. This is particularly valuable in showing the curves of intersection of surfaces and planes such as that illustrated in FS. 25. These film-strips should also be superior to blackboard drawings in showing how elements of area or of volume are used to build rectangles, columns or slices. The computation of the value of a triple integral cannot be done effectively by a film-strip. But just because it is included in the strip does not mean that it cannot be done on the board as usual so that students will follow each step. There will be disagreement with the statement that the fundamental problem of the Integral Calculus is that of finding areas bounded by curved lines.

Technical Qualities: Photography: Very good graphic drawings: FS. 25 is most effective with white on black. Content: Appropriate selection of curves, areas and volumes.

PICTURES

P. 2—*Descriptive Geometry Vectographs*

Society for Visual Education, Inc.; 100 East Ohio Street, Chicago, Illinois; Drawings by John T. Rule

Slides; fifty 2×2 slides; black and white; $62.50 per set

Description: The fifty three-dimensional vectographs in this set cover the following points:

1. Basic principles of orthographic projection and methods of obtaining auxiliary views.
2. True length and end view of a line, edge and normal view of a plane, and principles of perpendicularity.
3. Intersections of lines and planes and other problems involving planes.
4. Principles of rotation.
5. Point, line and plane problems using traces.
6. Intersections and developments of surfaces.
7. Principles of pictorial drawing.

Each slide states the principle illustrated and a two-dimensional view is placed beside the three-dimensional view. This enables the student to see how the planes in the three-dimensional view unfold to produce the two-dimensional view.

Appraisal: Projected on a metallic screen and viewed through a pair of simple Polaroid viewing spectacles, the three-dimensional quality of the pictures is so realistic that one feels that he is looking directly at a wire model placed in front of the screen. Perpendicular planes, which in conventional diagrams are indicated confusingly by lines crossing at an angle all in the one plane of the diagram, appear squarely perpendicular and clearly separated in space. The arcs subtended by angles between planes are immediately apparent. Since relationships of lines and angles are visible, the understanding of the student is quickened and teaching time shortened.

However, some of the drawings are very complex. To be used most effectively, it will be necessary for the student to see the slide at the time he is working on a similar drawing.

These slides may be projected by any standard projector for 2×2 slides. To achieve the three-dimensional effect, a metallic non-polarizing screen is necessary and each person must use a pair of three-dimensional viewers. These cost only eight dollars per hundred.

Technical Qualities: Content: Appropriate and varied, covering basic prin-

ciples. Drawings: Excellent. Print: Some variation in amount of three-dimensional effect.

SOURCES OF MATERIAL FOR LABORATORY WORK

SL. 4—*Slidecraft Lantern Slides*

Slidecraft Company; 257 Audley Street; South Orange, N. J.

Plastic slide Material; Two sizes: $2'' \times 2''$ and $3\frac{1}{4}'' \times 4''$; Small size: 100 for \$3 (with crayons \$3.50), 400 for \$10 (with crayons \$10.50). Large size: 25 for \$2 (with crayons \$2.50), 100 for \$6 (with crayons \$6.50), 1000 for \$55 (with 10 boxes of crayons \$60). Crayons alone: \$.60 per box, \$6 a dozen boxes.

Description: These plastic slides have one smooth side and one matted side; the edge is painted black. The matted surface can be drawn upon with pencil, pen or crayon, or tvpewritten on with or without a piece of carbon paper to intensify the image. The slide does not need to be enclosed between sheets of glass, but can be used and stored as it is. Colors are as successful as black and white.

Appraisal: The material is very durable, no heavier in weight than very light cardboard, and takes up no more room in storage. The making of slides with it is clean and rapid, and the projected image is clear. Images may be washed off for reuse of the slides, but this is not always entirely successful. For large review units, collections of historical material for supplementing text-books, complicated drawings and recreational material the slides would be very useful in mathematics. There is danger of enthusiasm suggesting that they replace a great deal of the present blackboard work; this is not at all desirable.

◆ THE ART OF TEACHING ◆

Impossible and Unsolved Problems in Elementary Mathematics

By Roy Dubisch

Triple Cities College of Syracuse University, Endicott, New York

Early in my teaching career I gave the following problem on a test in intermediate algebra:

A plane starts out from an airport traveling at 200 mph. Two hours later a second plane traveling at 300 mph leaves the same airport traveling in the same direction. At what time after leaving the airport will the second plane have traveled twice as far as the first?

A moment's reflection, of course, shows that the second plane can never have traveled twice as far as the first—even if the first plane did not have any head start at all! Thus my attempt to state a slight variation on the familiar "courier" problem was ill considered. The moral of my story, then, would seem to be only that one should be more careful in writing up test problems!

I gained a great deal more from giving this problem, however. For, in grading the test I found, on one paper after the other, the "solution"

$$\begin{aligned} 300\,t &= 2\,(200\,t)+400 \\ -100\,t &= 400 \qquad (1)\\ t &= -4 \end{aligned}$$

where t was taken to be the time elapsed after the second plane left the airport. Or, much too frequently,

$$\begin{aligned} 300\,t &= 2\,(200\,t)+400 \\ 100\,t &= 400 \qquad (2)\\ t &= 4 \end{aligned}$$

(with perhaps a slight smudge in front of the 100 where a minus sign had been erased!).

Now the first "solution," when properly interpreted, seems to me to be perfectly permissible (perhaps because it was in this way that *I* first noted that there was no solution to the problem!). For what it does, in effect, is to show us in a mechanical way that we have posed an impossible problem. However, not one student obtaining the solution (1) went on to say that the negative answer showed that the problem was impossible of solution. Much worse, of course, was the attitude of the other students who obtained "solution" (2) by (in most cases) deliberately violating the laws of algebra to get a solution which, superficially, seemed plausible.

I now believe that each student of algebra should be systematically exposed to "impossible" problems and be expected to recognise the impossibility of their solution by an algebraic analysis. That is; every class should have some work with problems of this type and be warned to expect an occasional problem of this type on a test.

Along with exposure to this type of problem which shows how algebraic analysis can show that no solution exists, I believe that the student should be severely penalized for obtaining an obviously ridiculous answer to a problem without commenting in some way on it. That is, while I realize, for example, that a very simple algebraic slip may result in the "n" of a problem involving integers coming out to be 15/13 and can easily condone the error, I do *not* condone the student's calmly presenting such a result as the solution *without comment.*

For example, in the problem first presented, I would not regard the solution (1)

as satisfactory unless the student added a comment such as: "The negative answer indicates that I have either made a mistake in my algebra or the problem has no solution. Since I have checked over my work and believe it to be correct I conclude that the problem is impossible." (If, in addition, he can detect the fallacy in the verbal statement of the problem, so much the better.) On the other hand, if he makes the same comment in regard to a ridiculous solution occasioned by faulty algebra I am still willing to give him some credit for the portions of his algebra that are correct—but no credit at all for such a solution without comment.

Such training in the recognition of an absurd answer is, I believe, far from impractical. In engineering and other fields where mathematics is used, one frequently asks the question: is a certain operation or structure possible?—and an answer to this question in the negative may be regarded as fully as important as an actual solution to another problem.

Not only should students be trained to detect algebraic solutions which indicate impossibilities but they should have some acquaintance with problems that cannot be solved with the mathematics they have had. I do not refer to extremely complicated problems whose solutions are obviously difficult but rather to deceptively simple sounding problems. For example, many problems involving trigonometry or simple calculus may appear, at first sight, to be capable of algebraic solution.

Finally, to conclude my plea for some work with impossible problems and comment on advanced problems, I would like to ask that every high school student be exposed to a discussion of the three classic impossibilities—the trisection of the angle, the duplication of the cube and the squaring of the circle as well as to some current unsolved (but not necessarily unsolvable) problems. In the latter category are Fermat's last theorem, Goldbach's conjecture and the four color map problem.* By such discussions we (1) increase the usefulness of mathematics by showing how it is used to detect impossibilities as well as possibilities, (2) show the necessity of further work in mathematics and (3) extend the student's mathematical horizon.

* See, for example Chap. VIII, "Impossibilities and Unsolved Problems" in *Fundamentals of Mathematics* by M. Richardson (Macmillan, 1947).

Notice to Readers of the Mathematics Teacher

Beginning October 1, 1948 the price of all Yearbooks of the National Council of Teachers of Mathematics has been increased to $3.00 each. This is due to the continued increase in cost of printing and publications of these volumes. We are sorry to have to make this announcement but we feel that the Yearbooks are well worth more than this amount to all of our readers.

This change applies to the 3d, 4th, 6th, 8th, 14th, 15th, 16th, 18th, 19th and 20th Yearbooks. All other Yearbooks are now out of print.

Please send all orders for the above to the Bureau of Publications, Teachers College, Columbia University, New York 27, New York. Please do not send orders to THE MATHEMATICS TEACHER.

EDITORIAL

The New President of the National Council of Teachers of Mathematics

Dr. Hildebrandt is a native of La-Salle, Illinois. His parents and grandparents had been missionaries on the Gold Coast of Africa. He was graduated from the Proviso Township High School in Maywood, Ill. (where Mr. E. W. Schrieber taught him 4th year mathematics) in 1918 at the age of 15. In 1922, he was graduated from the University of Chicago with the degree of Bachelor of Science, with honors in Mathematics. During the years 1924–31, while teaching, he did graduate work in mathematics and in education at the University of Chicago, and in mathematics of finance, insurance and statistics at the University of Michigan. He received his Ph.D. degree in June 1932 from the latter institution.

He was head of the Mathematics Department of the high school at Stevens Point, Wis. from 1922–24 and then, at the age of 21 became principal of the same school. He later taught in the mathematics departments of the University of Michigan, DePauw University, Brooklyn College, New Jersey State Teachers College at Montclair and since 1943 has been at Northwestern University where he is now an Associate Professor.

Dr. Hildebrandt's dissertation "Systems of Polynomials Connected with the Charlier Expansions and Pearson's Differential and Difference Equations" was published in 1931 in *Annals of Mathematical Statistics*. He was associate Editor of *American Mathematical Monthly* in charge of Mathematics Clubs Department from 1938–1941.

He joined the National Council of Teachers of Mathematics in 1932 and was chairman of the Multi-Sensory Aids Committee, 1940–1945, the report of which committee was published as the 18th Yearbook of the NCTM. He was a member of the Board of Directors from 1944–47 and Second Vice President, 1947–48. He is also a member of the American Mathematical Society, of which he has been on the Board of Governors since 1947; of the Institute of Mathematical Statistics; American Association for the Advancement of Science (since 1944 he has been a representative of NTCM on the Cooperative Committee of the AAAS on Teaching of Science and Mathematics); Central Association of Science and Mathematics Teachers, of which he has been secretary of the mathematics section since 1947; Men's Mathematics Club of Chicago, Association of Mathematics Teachers of New Jersey; and the National Science Teachers Association. He has been a member of Examiners in Mathematics, College Entrance Examination Board since 1946. His fraternities are Sigma Xi; Pi Mu Epsilon, national honorary mathematics fraternity in which he has held several offices and is now Secretary-Treasurer General; and Kappa Mu Epsilon, national honorary mathematics fraternity.

The Mathematics Teacher wishes to congratulate both Dr. Hildebrandt and the Council on his election to the presidency and to wish him the greatest success.—W. D. R.

IN OTHER PERIODICALS

By NATHAN LAZAR

Midwood High School, Brooklyn 10, New York

American Mathematical Monthly

April 1948, Vol. 55, No. 4

1. May, Kenneth, "Probabilities of Certain Election Results," pp. 203–209.
2. Bunch, W. H., "The Quadrilaterals of Pascal's Hexagram," pp. 210–217.
3. Court, N. A., "Notes on Cospherical Points," pp. 218–221.
4. Schorling, Raleigh, "A Program for Improving the Teaching of Science and Mathematics," pp. 221–237.
5. Mathematical Notes, pp. 238–241.
   a. Caris, P. A., "Rational Solutions of a Diophantine Equation."
   b. Kuo, Huang-Ting, "On a Contact Transformation Theorem."
6. Classroom Notes, pp. 241–247.
   a. Pólya, George, "Generalization, Specialization, Analogy."
   b. Pang, Hoi-Cheung, "Areas of Plane Figures."
7. Elementary Problems and Solutions, pp. 248–252.
8. Advanced Problems and Solutions, pp. 253–259.
9. Recent Publications, pp. 260–264.
10. Club and Allied Activities, pp. 265–267.
11. News and Notices, pp. 267–270.
12. Official Reports and Communications, pp. 270–276.

Bulletin of the Kansas Association of Teachers of Mathematics

April 1948, Vol. 22, No. 4

1. Edington, Will E., "Teaching Mathematics as a Guide to Practical Living," pp. 51–54.
2. Norris, Ruby, "The Use of Puzzles and other Recreational Aids in the Teaching of Mathematics," pp. 55–56.
3. Betz, William, "Elements of a Functional Program in Mathematics," pp. 56–60.
4. Sanger, R. G., "History of Geometry," pp. 60–62.

The Duodecimal Bulletin

March 1948, Vol. 4, No. 1

1. McLelland, Nina, "An Ideal Numerical Base," pp. 1–4.
2. The Annual Meeting, pp. 5–8.
3. "Four Fours," pp. 8–9.
4. The Annual Award, pp. 10–11.
5. Elbrow, G., "The New English System of Money, Weights, and Measures and of Arithmetic," pp. 11–23.
6. H. C. R., "An Aid to Calculating $1/n$," pp. 24–26.
7. More Jr., T., "Magic Square," p. 27.
8. "The Mail Bag," p. 28.
9. "Counting in Dozens," p. 29.

Mathematical Gazette

February 1948, Vol. 32, No. 298

1. Annual Meeting, p. 1.
2. Report of the Council, pp. 2–5.
3. Memorandum to the Minister of Education, p. 5.
4. Jeffrey, G. B., "Mathematics as an Educational Experience," pp. 6–14.
5. Watson, G. N., "A Comedy of Errors," pp. 15–16.
6. "A Unified Course in Mathematics in Secondary Schools," pp. 17–36.
7. Mathematical Notes, pp. 37–39.
8. Reviews, pp. 40–48.

BOOK REVIEWS

Surveying Instruments—Their History and Classroom Use. By Edmond R. Kiely, Ph.D. Bureau of Publications, Teachers College, Columbia University, New York, 1947. 411 pages. $3.00.

Few teachers can afford to buy the many texts on their own or related subjects that are constantly coming from the printing presses, however, every now and then a really great book that no teacher can afford to do without appears. Such a book is Surveying Instruments by Dr. Edmond R. Kiely.

During the last twenty years there has been a constantly increasing interest in mathematical instruments and the type of mathematics known as Field Work. There is every reason to believe that this interest will continue, that the simpler instruments will find wide use in arithmetic, algebra, and geometry classes and that the level, transit, and sextant will become a regular part of every course in trigonometry or advanced general mathematics.

In the first 238 pages Dr. Kiely traces the history of all the early measuring and surveying instruments. It is difficult to realize the enormous amount of research that this entailed. The difficulty of Dr. Kiely's task was increased by the fact that many of the original texts used by him were in Latin, French, German or other foreign languages.

All of the material in the first 238 pages is of interest to teachers of mathematics engineering and science and in no other place can this material be obtained.

The last 121 pages of the Dr. Kiely's book is devoted to the classroom use of the various instruments described and illustrated in the first 238 pages. Dozens of interesting exercises that may be used in the teaching of mathematics are given illustrated and explained. One may be confident that the authors of mathematics texts will borrow very heavily from this book and that it will have a pronounced effect on the teaching of mathematics.

Surveying Instruments is unusually well illustrated. It has more than 200 pictures many of them full pages reproductions from the original early texts. These have increased the cost of printing the book but have immeasurably increased the value of the text to both teacher and pupil. From the picture the construction and use of most of the instruments is immediately apparent. The pictures will be a great help to teachers who wish to construct and use models of these early instruments.

The most interesting feature of Surveying Instruments is the fact that it is ageless. Years from now long, long after the rather limited edition is exhausted individuals and libraries will be paying premium prices for the few copies that reach the old book lists.—C. N. SHUSTER.

College Algebra. By Frederick S. Nowlan. McGraw-Hill, N. Y. xiv+371 pages. Price $3.00.

There is an increasing tendency in recent text books to use algebra to teach mathematical thinking, which function has been traditionally assigned mainly to geometry. This Canadian text makes a worthy contribution to the trend. In the preface Dr. Nowlan states: "Emphasis is placed upon the concept of algebra as a postulational science." This emphasis is most evident in two places. First, a critical review of elementary and intermediate algebra. Second, an introduction to complex numbers as ordered pairs of real numbers. This treatment removes the mystery usually surrounding the "invention" of the imaginary unit and provides a clear illustration of the constructive method of extending a number of system.

The book covers all of the standard topics of college algebra. The definitions are unusually careful and the author indicates those places where proofs are omitted because of excessive difficulty. The explanations are easy to follow, numerous examples being worked out completely as illustrations.—FREDERIC W. BORGES.

Plane Geometry. By Daniel T. Sigley and William T. Stratton. Dryden Press, N. Y. 242 pages. Price $2.25.

This text was not designed for the standard tenth-grade course in demonstrative geometry. It presupposes only one year of high-school algebra, but requires more mathematical maturity than is possessed by most tenth-grade students. Intended primarily for colleges, it is worth trying as early as the eleventh grade with students who have completed one and a half or two years of algebra.

The selection of topics, organization, and style are original and excellent. Features which merit special praise include the extensive use of the principle of necessary and sufficient conditions, the early treatment of proportion and similarity, the inclusion under locus of conics and linkages, and the section on modern geometry.

The virtual elimination of the divided-page format in favor of the modern discussion-type proof is perhaps the most significant departure from tradition. This, together with the numerous practical applications, brings the book closer to the spirit of living mathematics than to that of ancient Greek geometry.

On the debit side are a few small logical errors, which can be easily corrected, however, by the alert teacher. The use of superposition as a method of proof is a disappointing note in an otherwise progressive work.—FREDERIC W. BORGES.

The National Council of Teachers of Mathematics Ninth Christmas Conference

Ohio State University
Columbus, Ohio

December 29 and 30, 1948

THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS will hold its ninth Christmas Conference at Ohio State University, Columbus, Ohio, on Wednesday and Thursday, December 29 and 30, 1948 During this same week, meetings of the American Mathematical Society and the Mathematical Association of America will also be in session on the Ohio State University campus.

Headquarters for the National Council will be in Baker Hall on the University campus. Six sectional meetings will be held in this building and smaller discussion groups will meet on Thursday morning in Derby Hall, a classroom building nearby. Professor Harold Fawcett of the College of Education is local chairman for this Conference.

All members of the National Council will be housed in Baker Hall. The majority of rooms are double rooms and the fee for these rooms is $2.00 a day per person. A limited number of single rooms are available at $2.50 per day.

Meals may be obtained in Baker Hall at an overall club rate of $2.20 a day or at individual meal rates as follows: breakfast, 50¢; lunch, 80¢; and dinner $1.15. There will be a banquet on Wednesday evening.

Those desiring hotel accommodations should make their reservations directly with the hotel management. Hotels and their prices for single and double rooms are as follows.

	Single	Double
Chittenden Hotel	$2.75–3.75	$5.00– 9.00
Deshler-Wallick	4.50–9.00	7.00–15.00
Fort Hayes Hotel	4.00–5.50	7.00– 8.00
Neil House	2.00–7.00	4.00– 7.00
Seneca Hotel	3.00–4.50	5.00– 6.50
Southern Hotel	3.00–7.00	4.50– 7.00

Reservations for rooms and meals in Baker Hall should be mailed as soon as possible. The following information should be sent to Mr. Oscar Schaaf, Room 120 Arps Hall, Ohio State University, Columbus, Ohio, not later than December 15, 1948.

1. Nights for which room reservations are desired: Tuesday night, Wednesday night, Thursday night.
2. Since most rooms are arranged for double occupancy, include name of person for second reservation, if possible.
3. List meals you wish to take in the Baker Hall Dining room: Tuesday dinner; Wednesday breakfast and luncheon; Thursday breakfast, luncheon and dinner.
4. Do you wish a reservation for the banquet on Wednesday evening?

An exhibit of Mathematical Models will be located in the Social Room of Baker Hall. Teachers are invited to bring materials for demonstration. Communications regarding exhibit space should be addressed to Professor Harold Fawcett, Ohio State University.

PROGRAM

WEDNESDAY, DECEMBER 29, 1948

9:00 A.M. Mathematics Films—Baker Hall, West Dining Room
Periodic Functions
Principles of Measure

10:00 A.M. High School Section—Baker Hall, West Dining Room
Theme: "Statistics at Work in Industry"
Presiding: E. H. C. Hildebrandt, Northwestern University
Demonstration of Applications of Shewhart Control Charts in Statistical Quality Control
Lloyd A. Knowler, University of Iowa
Demonstration of the Principles of Modern Acceptance Sampling Science
Mason E. Wescott, Northwestern University

10:00 A.M. Junior College Section—Baker Hall, East Dining Room

Presiding: J. O. Hassler, University of Oklahoma

Comments on the Contents of and Teaching Techniques Used in Courses in Elementary Collegiate Mathematics

R. G. Sanger, Kansas State College

A Mathematics Testing Program

M. W. Keller, Purdue University

Objectives in the Teaching of Collegiate Mathematics

F. S. Nowlan, University of Illinois, Navy Pier, Chicago

1:15 P.M. Mathematics Films—Baker Hall, West Dining Room

The Language of Graphs

Installment Buying

Geometry and You

2:00 P.M. High School Section—Baker Hall, West Dining Room

Presiding: H. W. Charlesworth, East High School, Denver, Colorado

Developing Interest in General Mathematics

Philip Peak, University School, Indiana University

Teaching for Generalization in Geometry

Frank B. Allen, Lyons Township High School, LaGrange, Illinois

Objectives and Programs for a High School Mathematics Club

K. C. Schraut, University of Dayton

2:00 P.M. In-service Training of Teachers—Baker Hall, East Dining Room

Presiding: Vera Sanford, State Teachers College, Oneonta, New York

How a State Association of Mathematics Teachers Can Contribute to the Development of Teachers

Gilbert Ulmer, University of Kansas

Mathematics Institutes

W. W. Rankin, Duke University

Conferences on the Teaching of Elementary and Secondary Mathematics

Bjarne R. Ullsvik, Illinois State Normal University, Normal, Illinois

4:00 P.M. Mathematics Films—BakerHall, West Dining Room

6:00 P.M. Banquet—Baker Hall, East Dining Room

Thursday, December 30, 1948

9:00 A.M. Mathematics Films—Room 108, Derby Hall

What is Algebra?

Per Cent in Every Day Life

Measurement

10:00 A.M. Discussion Groups and Clinics

***NOTE:* Reservations for attendance at Discussion Groups and Clinics must be made before December 15, 1948. First, Second and Third Choices of Groups should be sent to: National Council of Teachers of Mathematics, 212 Lunt Building, Northwestern University, Evanston, Illinois.**

GROUP I—Room 100 A Derby Hall

Leader: M. H. Ahrendt, Anderson College, Anderson, Indiana

Topic: "How can we Teach for Understanding in Mathematics?"

Group II—Room 101 Derby Hall

Leader: Lee E. Boyer, State Teachers College, Millersville, Pennsylvania

Topic: "What Simple Applications of Algebra should be used to Coordinate the Teaching of Algebra with Arithmetic?"

Group III—Room 102 Derby Hall

Leader: Irving W. Burr, Purdue University

Topic: "What Principles and Applications of Statistics should be taught in the High School and in the Junior College?"

Group IV—Room 103 Derby Hall

Leader: H. W. Charlesworth, East High School, Denver, Colorado

Topic: "How can we Provide Better Coordination between our High School and College Mathematics Programs?"

Group V—Room 103 A Derby Hall

Leader: H. E. Grime, Supervisor of Mathematics, Cleveland, Ohio

Topic: "What Provisions should be made for Individual Differences in the Study of Algebra and Geometry?"

Group VI—Room 104 Derby Hall

Leader: J. O. Hassler, University of Oklahoma

Topic: "How shall we Teach Demonstrative Geometry to Secure the Maximum Transfer Value?"

Group VII—Room 105 Derby Hall

Leader: Gertrude Hendrix, State Teachers College, Charleston, Illinois

Topic: "A Sensitiveness to Inconsistency: Born? Made? or Sprouted in a Plane Geometry Class?"

Group VIII—Room 105 A Derby Hall

Leader: Phillip S. Jones, University of Michigan

Topic: "What Type of Mathematics should be Included in the Gen-

eral Mathematics Courses in College?"

Group IX—Room 106 Derby Hall
Leader: H. T. Karnes, Louisiana State University
Topic: "What Methods and Materials can be used to improve the teaching of Trigonometry and College Algebra?"

Group X—Room 107 Derby Hall
Leader: Sheldon S. Myers, Western State High School Kalamazoo, Michigan
Topic: "What are Some Industrial and Scientific Applications of Secondary Mathematics Appropriate for High School Students?"

Group XI—Room 108 Derby Hall
Leader: Henry W. Syer, Boston University
Topic: "What is Wrong with Films and Filmstrips in Mathematics and What Improvements should be Made in Them?"

Group XII—Room 109 Derby Hall
Leader: John F. Schacht, Bexley High School, Columbus, Ohio
Topic: "How can Models and Flexible Devices be Used to Vitalize Geometry Instruction?"

Group XIII—Room 200 Derby Hall
Leader: Carl N. Shuster, State Teachers College, Trenton, N. J.
Topic: "Computation with Approximate Numbers"

Group XIV—Room 202 Derby Hall
Leader: Norma Sleight, New Trier Township High School, Winnetka, Illinois
Topic: "What Problems and Supplementary Material can be Used Most Effectively in the Teaching of Graphing of Second Degree Equations?"

Group XV—Room 204 Derby Hall
Leader: Bjarne R. Ullsvik, Illinois State Normal University, Normal, Illinois
Topic: "Constructing Tests in Algebra and Geometry."

Group XVI—Room 207 Derby Hall
Leader: E. A. Whitman, Carnegie Institute of Technology
Topic: "Visual Aids for the Junior College Teacher of Mathematics."

Group XVII—Room 208 Derby Hall
Leader: James H. Zant, Oklahoma A. and M. College
Topic: "The Content of High School Mathematics Courses with Reference to Two Types of Students: Those Preparing to Study Science, Mathematics or Engineering in College, and Those who need Training for Citizenship Only."

2:00 P.M. High School Section—Baker Hall, West Dining Room
Theme: "Mathematical Preparation for Science and Engineering"
Presiding: James H. Zant, Oklahoma A. and M. College

A Report on the Mathematical Deficiencies of High School Students Entering Engineering Colleges
Frederic H. Miller, School of Engineering, Cooper University for the Advancement of Science and Art

Applications of Mathematics in Pre-Engineering Mathematics
John W. Cell, North Carolina State College

Fundamentals and Engineering Mathematics
Kaj L. Nielsen, Naval Ordnance Plant, Indianapolis, Indiana

The Engineer Looks at High School Mathematics
(Speaker to be Announced)

2:00 P.M. Teacher Training Section—Baker Hall, East Dining Room
Presiding: Carl N. Shuster, State Teachers College, Trenton, N. J.

Certification and Professional Standards for Teachers of Mathematics
Howard F. Fehr, Teachers College, Columbia University

Would Contests and Scholarships Contribute to Increased Interest in Mathematics?
John Mayor, University of Wisconsin

The Need for Cooperative Effort in Teacher Training
F. Lynwood Wren, George Peabody College for Teachers

4:00 P.M. Films—Baker Hall, East Dining Room

LIST OF MATHEMATICS CLUBS (By States)

ARKANSAS

Mathematics Science Council
Chairman: Miss Kathryn Buchanan, c/o Senior High School, Fort Smith, Ark.

CALIFORNIA

California Mathematics Council
President: Don Kaufman, Sequoia High School, Redwood City, Calif.
Exec. Sec.: Mrs. Ruth G. Sumner, Oakland High School, Oakland 10, Calif.

COLORADO

Mathematics Club
President: Vernon Hoxie, Denver, Colo.
Faculty Sponsor: Miss Lillian E. Duer, 4201 Raleigh St., Denver 12, Colo.
Mathematics Section of Colorado Education Assoc.
President: Dr. A. W. Recht, Univ. of Denver, Denver, Colo.

CONNECTICUT

Lambda Mu Nu—Teachers College of Conn.
President: Henry Passerini, 46 Erwin Pl., New Britain, Conn.

D.C.

William Wallis Mathematics Club
Adviser: Miss Veryl Schult, Wardman Park Hotel, Washington, D.C.

DELAWARE

Mathematics Club of Univ. of Delaware
President: Mary Jane Smith, University of Delaware, Newark, Del.
Secretary: James Goldey, University of Delaware, Newark, Del.
Alpha Chapter of Delaware Pi Mu Epsilon
President: Carl Nelson, University of Delaware, Newark, Del.
Secretary-Treasurer: R. W. Jones, University of Delaware, Newark, Del.

FLORIDA

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Dade County Junior High Mathematics Teachers Association
President: Mrs. Alma L. Stoetereau, 757 N.E. 72 Ter., Miami 38, Fla.
Secretary: Mrs. Nell Grigsby, 1799 N.E. 4th Ave., Miami, Fla.

GEORGIA

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Indianapolis Mathematics Club
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Secretary: Mr. William Huxford, Rosedale, Ind.
Junior High Mathematics Club
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IOWA

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Iowa Assoc. of Mathematics Teachers
President: Nona Moss, Fort Dodge, Iowa
Secretary-Treasurer: Ruth Smith, Esterville, Iowa
Mathematics Club
President: Orval Knee, Box 157, Seerley Hall, Cedar Falls, Iowa
Secretary: Marilyn Schmidt, Lawrence Hall, Cedar Falls, Iowa
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President: George Mack, 415 W. 7th, Cedar Falls, Iowa
Secretary: Lena Abbas, Lawther Hall, Iowa State T.C., Cedar Falls, Iowa
Pi Mu Epsilon (Alpha)
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Kansas

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Wichita Mathematics Assoc.
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Kansas Assoc. of Teachers of Mathematics
President: Miss Florence Ross, Ellsworth, Kan.
Secretary: Martha Rayhill, 2042 Massachusetts St., Lawrence, Kan.

Kappa Mu Epsilon—Honorary Math. Fraternity
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Corres. Sec.: Charles B. Tucker, Kansas State T.C., Emporia, Kan.

Mathematics Club
President: Leo J. McCue, 805 Market, Emporia, Kan.
Secretary: Lorena B. Adam, 1016 Constitution, Emporia, Kan. (Kansas State Teachers College)

Mathematics For Fun
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Kentucky

Conference of Mathematics Teachers
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Secretary: Mr. Willard C. Taylor, 1464 S. Third St., Louisville, Ky.

Mathematics Forum
President: Mrs. James Taylor, 152 Crestwood, Louisville, Ky.

Louisiana

Tulane Mathematics Club
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Louisiana-Mississippi Section of the National Council of Teachers of Mathematics
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Maryland

Mathematics Section of Maryland State Teachers Association
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Massachusetts

The Association of Teachers of Mathematics in New England
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Michigan

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Detroit Mathematics Club
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The Minneapolis Mathematics Club
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Secretary: Miss Alice Hansen, 809 W. 53rd St., Minneapolis 9, Minn.

Mississippi

La.-Mississippi Sect. of National Council of Teachers of Math.
Listed under Louisiana

Missouri

St. Louis District Council of Teachers of Mathematics
President: Mr. William D. Christian

Kappa Mu Epsilon, Missouri, Delta Chapter
The University of Kansas City
President: Karl Eaton, 412 W. 47th St., Kansas City
Secretary: Elizabeth Sullivan, 2544 Mersington, Kansas City

Nebraska

The Nebraska Section of the National Council of Teachers of Mathematics
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New Hampshire

The Southeastern New Hampshire Assoc. of Mathematics Teachers
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New Jersey

Kappa Mu Epsilon (Honorary)
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OKLAHOMA

Oklahoma Council of Teachers of Mathematics
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OREGON

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PENNSYLVANIA

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Mathematics Department Meeting
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Pi Mu Epsilon, Delta Chapter of Pennsylvania
Faculty Adviser: W. J. Harrington, Sparks Bldg.
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Secretary: Mrs. Alice Scallon, Pennsylvania State College
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Mathematics Section Meeting
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President: Seaton E. Anderson, 1043 W. 41st, Houston, Tex.